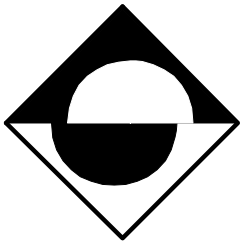
Nonlinear Model Predictive Control of Retrofit Heat-Exchanger Networks

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ABSTRACT

- Control of a retrofit Heat-Exchanger Network (HEN) was studied within the framework of Nonlinear Model Predictive Control (NMPC).

- For this purpose, an optimal retrofit design of the HEN was performed, and thus, bypasses as control variables were placed in order to satisfy steady-state hard/soft target-temperature constraints.

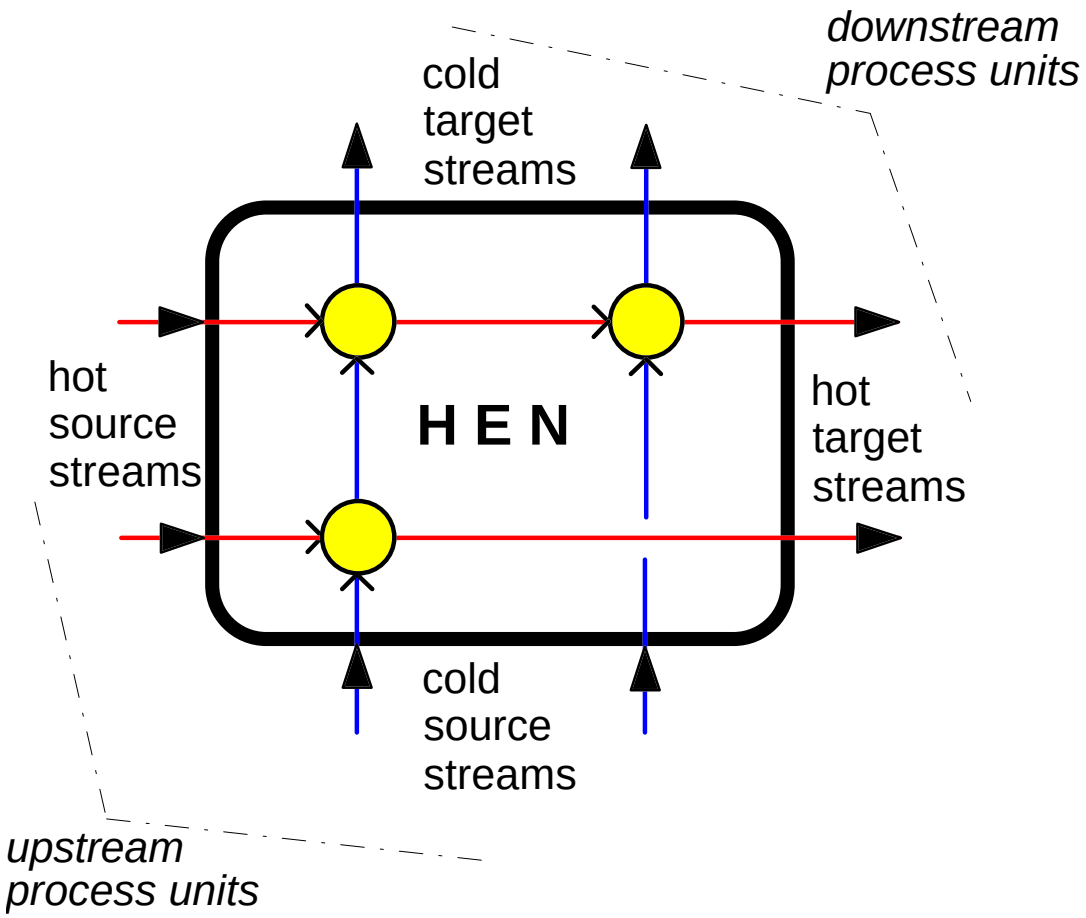
- Control configuration was determined and tested under decentralized feedback-control scheme which did not satisfy the constraints.

- The NMPC scheme, in which the nonlinear distributed-parameter HEN model was solved sequentially, satisfied all the temporal and steady-state hard/soft target-temperature constraints.

- A version of the classical NMPC scheme, in-lined with piecewise-continuous control-vector parameterization, showed a superior performance with regard to smoothness of the response and significant reduction in computation time.

Keywords: retrofit HEN, NMPC, NMPC with piecewise-continuous control-vector parameterization

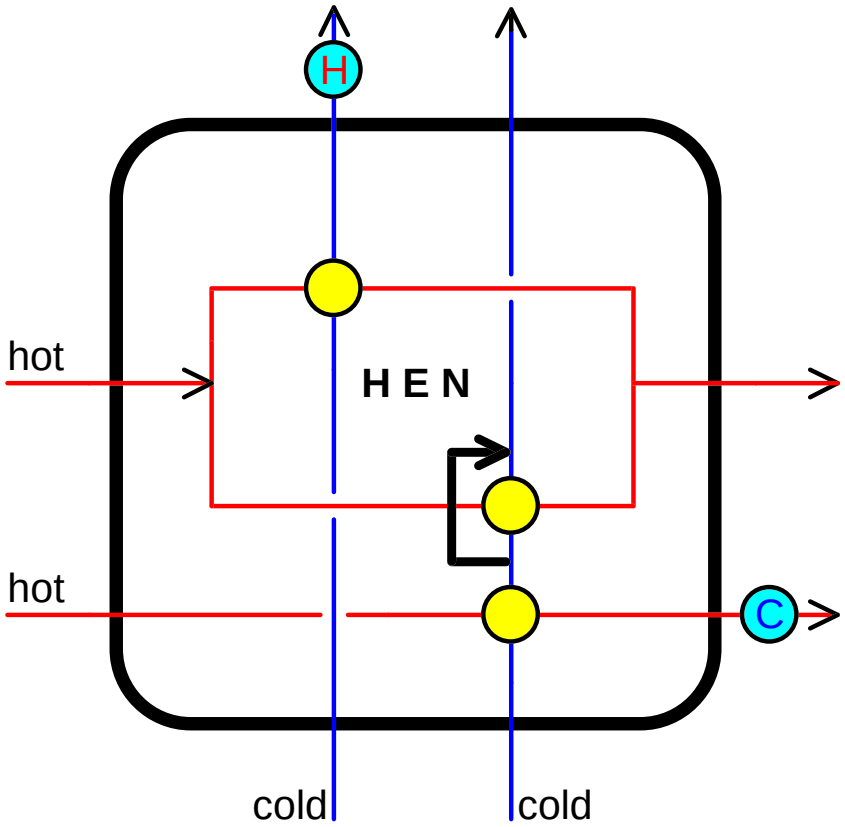
CONTROL PROBLEM



- The major problem in control of HENs is the lack of degrees of freedom, i.e., there may not be enough nu mber of variables to manipulate in order to achieve control objectives.

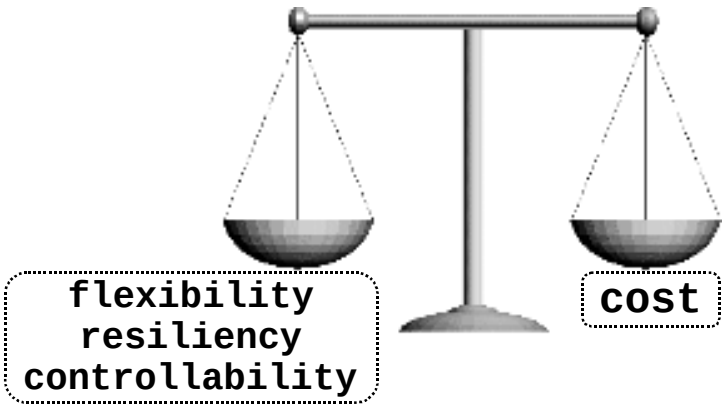
- Static HEN synthesis methods do not consider bypass of one or multiple exchangers and gene rally yield structures which do not contain any or enough number of stream splits.

RETROFIT-DESIGN PROBLEM



- The manipulatable variables (heater or cooler loads, stream-split ratios) that may be present in a HEN may not be sufficient to achieve the desired control performance.
- Hence, incorporation of one or more bypass streams is necessary to broaden controllable range of a HEN.
- However, the addition of bypasses requires updating the individual exchanger areas and/or utility exchanger loads. Therefore, it is necessary to re-evaluate the cost of a retrofit HEN containing bypass streams.

OPTIMAL RETROFIT-DESIGN



- Location and number of bypass streams affect:
  - investment cost (total area)
  - operating cost (utility)
  - flexibility (static-resiliency)
  - dynamic-resiliency
  - disturbance propagation paths
- There exists a trade-off between controllability (disturbance-rejection ability) and cost (area/utility) of a HEN.

OPTIMAL RETROFIT-DESIGN PROBLEM

Given:

- HEN structure
- Nominal operating conditions
- Target-temperature specifications as:
  - hard constraints (± 0 deviation)
  - soft constraints ($\pm \Delta T$ deviation)
- Locations of bypass streams

Obtain:

- New (incremental) exchanger areas
- Nominal values of bypass fractions

Such that:

- The retrofit HEN remains feasible:
  - for the nominal operating conditions
  - under all possible + or – directed disturbances of pre-defined (expected) magnitudes

Optimization:

- Minimum cost (total network area) design
- Maximum flexibility in each disturbance direction

FORMULATION OF OPTIMAL RETROFIT-DESIGN PROBLEM

objective function:

$$\underset{S^k,\,\mathbf{u}^j,\,\mathbf{A}}{maximize}\left\{\sum_k S^k \Big/ \sum_n A_n\right\} \qquad k=1,\ldots,N_d \quad j=0,\ldots,N_d$$

equality constraints:

$$\mathbf{T}^{t,k} - \overline{\mathbf{T}}^{t} = 0 \qquad k=0$$

inequality constraints:

$$\overline{\mathbf{T}}^{t} - \Delta\mathbf{T}^{t} \le \mathbf{T}^{t,k} \le \overline{\mathbf{T}}^{t} + \Delta\mathbf{T}^{t} \qquad k=1,\ldots,N_d$$

$$(1-r)\mathbf{u}^{0} \le \mathbf{u}^{j} \le (1+r)\mathbf{u}^{0} \qquad j\neq 0 \quad k=1,\ldots,N_d$$

side constraints:

$$u^{min} \le \mathbf{u}^{j} \le u^{max} \qquad j=0$$

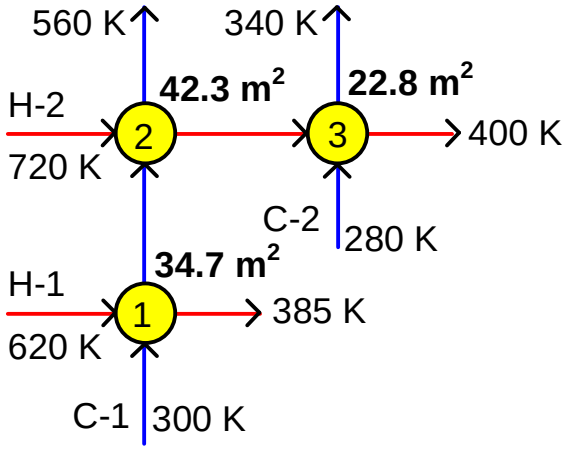
S^k (static flexibility): magnitude of the maximum disturbance (e.g., ± directed changes in source temperatures) that a structure can tolerate at steady state within prescribed operational limits (hard/soft target-temperature constraints).

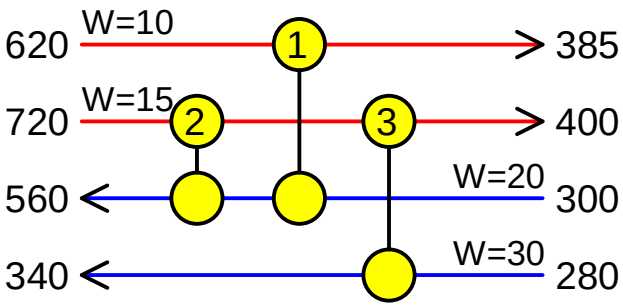
EXAMPLE HEN

Definition of Example HEN Problem (U_n = 0.5 kW/m^2-K)

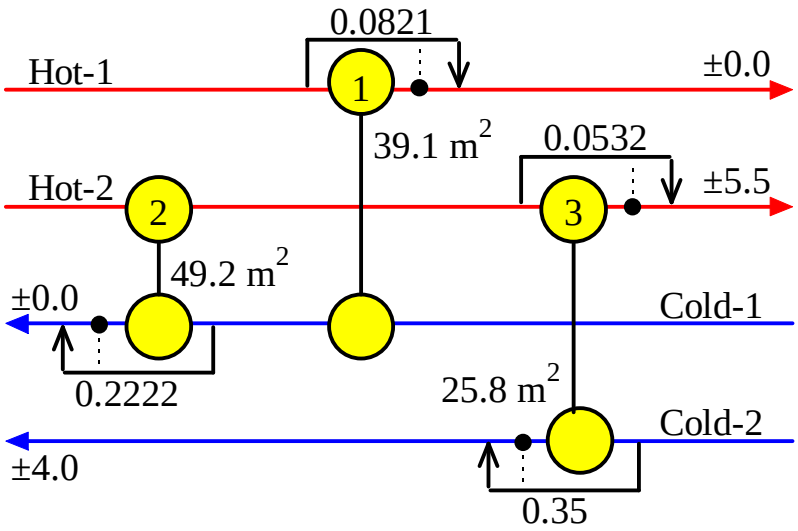
Stream	W (kW/K)	T_{in} (K)	T_{out} (K)
Hot-1	10	620	385
Hot-2	15	720	400
Cold-1	20	300	560
Cold-2	30	280	340

ORIGINAL HEN





RETROFIT HEN



DYNAMIC MODEL OF EXCHANGERS

- Single-tube-pass, shell-and-tube exchangers.
- Design parameters were evaluated using a detailed design algorithm.
- Distributed-parameter model of the heat exchangers were used to construct the dynamic model of the HENs by connecting the PDEs of the individual exchangers through boundary conditions.
- Utilizing the “method of lines” numerical technique, these PDEs were discretized in z-direction, and thus, converted into ODEs using “five-point biased-upwind” finite-difference formulas, which *eliminated steady-state offset* problem.

$$\frac{\partial T_H}{\partial t} = -v_H \frac{\partial T_H}{\partial z} + \frac{h_H a_H h_C a_C}{h_C a_C + h_H a_H} \frac{(T_C - T_H)}{A_H \rho_H C_{p_H}}$$

$$\frac{\partial T_C}{\partial t} = +v_C \frac{\partial T_C}{\partial z} + \frac{N_T h_C a_C h_H a_H}{h_C a_C + h_H a_H} \frac{(T_H - T_C)}{A_C \rho_C C_{p_C}}$$

bypass mixing cold side: $T_C^t = T_C^o u_C + T_C(1-u_C)$

bypass mixing hot side: $T_H^t = T_H^o u_H + T_H(1-u_H)$

STEADY-STATE OPTIMAL CONTROL OF RETROFIT HENs

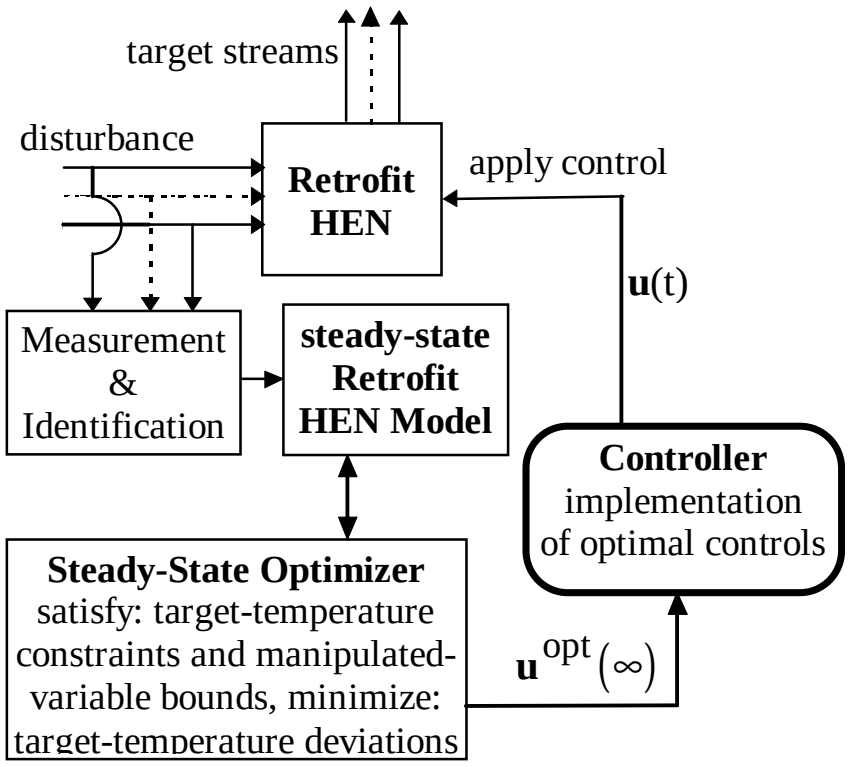
The static optimization problem can be defined as:

- When the HEN experiences disturbances in source-stream temperatures (or flowrates),
  - *find set of bypass fractions* that will minimize total deviations in the target temperatures,
  - subject to steady-state *target-temperature constraints* (soft / hard) and *manipulated-variable bounds*, such that the HEN remains feasible.

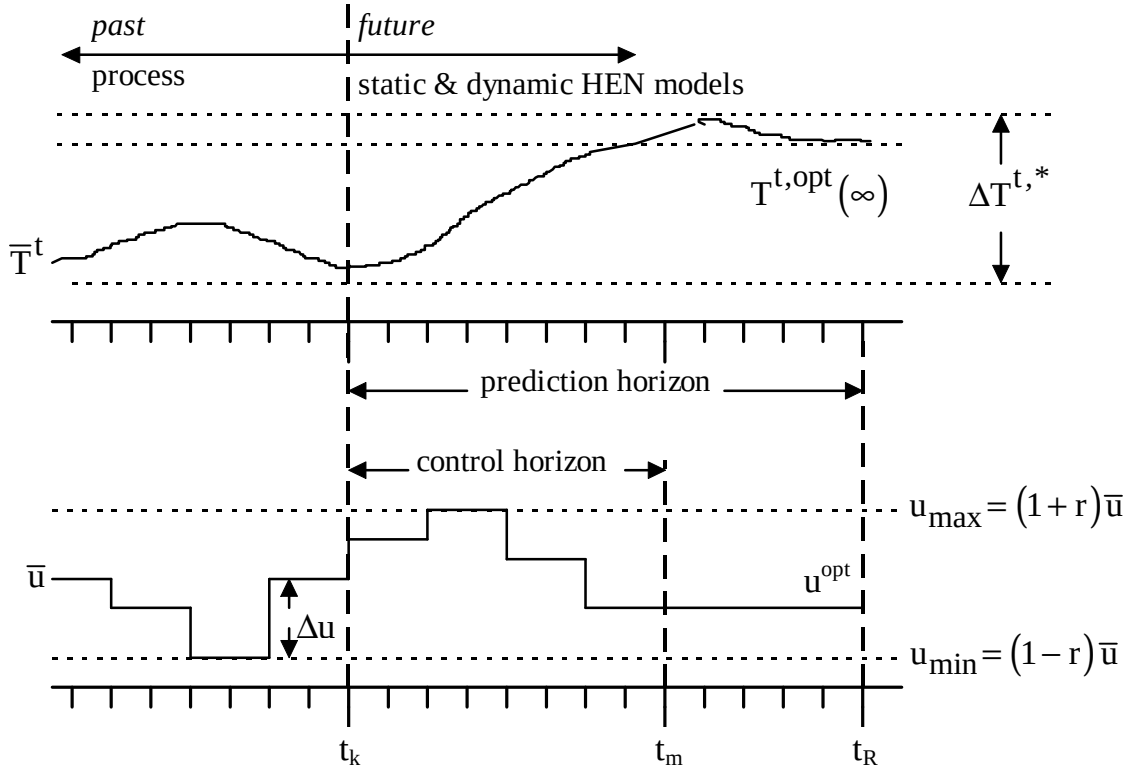
objective function: $\underset{\mathbf{u}}{minimize} \quad \left\| \Delta \mathbf{T}^{t} \right\|$

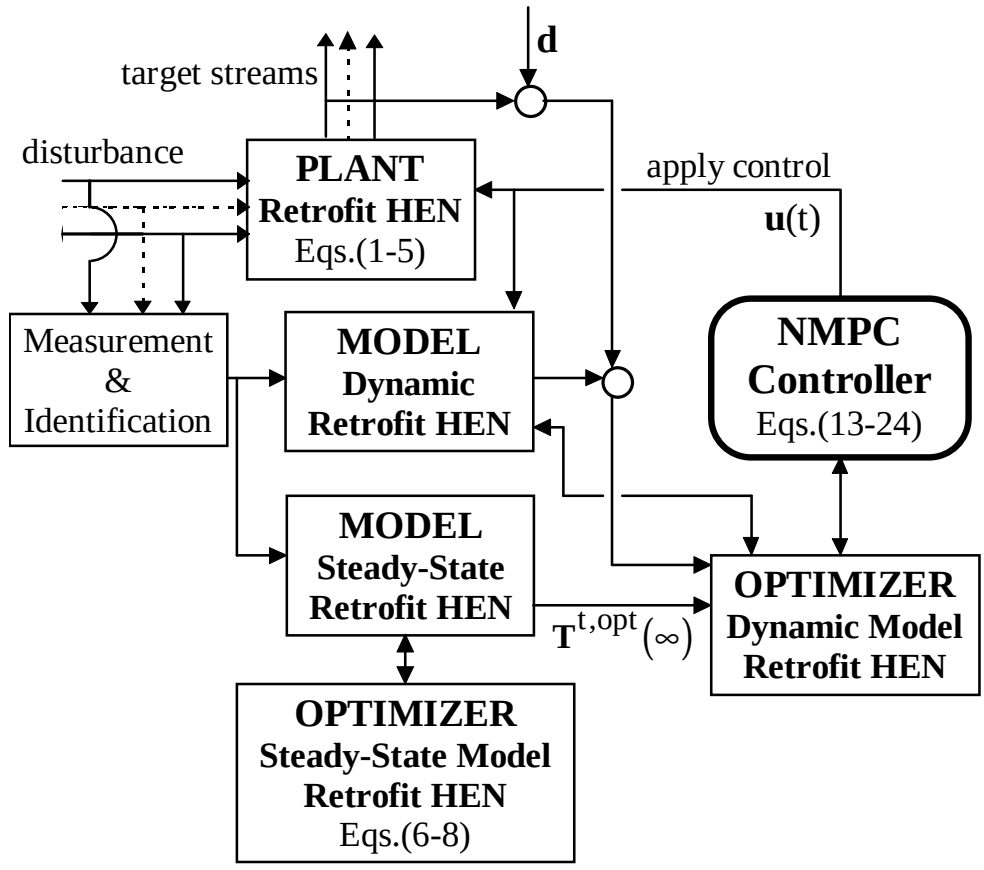
soft/hard target constraints: $\Delta \mathbf{T}^{t}_{min} \le \Delta \mathbf{T}^{t} \le \Delta \mathbf{T}^{t}_{max}$

manipulated-variable bounds: $(1-r)\mathbf{u}^{0} \le \mathbf{u} \le (1+r)\mathbf{u}^{0}$



NMPC OF RETROFIT HENs





FORMULATION OF NMPC FOR RETROFIT HENs

$$\underset{\mathbf{u}(i),\ldots,\mathbf{u}(i+m-1)}{\textit{minimize}} \quad \Phi(\mathbf{u}(i)) = \int_{t_i}^{t_{k+R}} \left\| \mathbf{T}_k^{t,opt}(\infty) - \mathbf{T}_{MP}^{t}(i) \right\| dt \qquad i=k,\ldots,k+R$$

$$\mathbf{T}_{MP}^{t}(i) = \mathbf{T}_{M}^{t}(i) + \mathbf{d}(k) \qquad i=k,\ldots,k+R$$

$$\mathbf{d}(k) = \mathbf{T}_{P}^{t}(k) - \mathbf{T}_{M}^{t}(k)$$

$$\dot{\mathbf{x}}(t,z) = \boldsymbol{f}\left(\mathbf{x}(t,z), \partial\mathbf{x}(t,z)/\partial z, \mathbf{u}(t), \mathbf{p}, \mathbf{d}\right) \qquad t \in [k,\ldots,k+R]$$

$$\mathbf{x}(k,z) = \bar{\mathbf{x}}(z)$$

$$\Delta\mathbf{T}_{min}^{t,*} \le \left\{ \mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} \right\} \le \Delta\mathbf{T}_{max}^{t,*} \qquad t \in [k,\ldots,k+R]$$

$$(1-r)\bar{\mathbf{u}} \le \mathbf{u}(i) \le (1+r)\bar{\mathbf{u}} \qquad i=k,\ldots,k+R$$

$$-r^{*}\mathbf{u}(i) \le \Delta\mathbf{u}(i+1) \le r^{*}\mathbf{u}(i) \qquad i=k+1,\ldots,k+R$$

$$\Delta\mathbf{u}(i+1) = \mathbf{u}(i+1) - \mathbf{u}(i)$$

$$\mathbf{u}(i) = \mathbf{u}(k+m-1) \qquad \forall\ i > k+m-1$$

$$\dot{x}(t) = \left\| min\left\{0, \left(\mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} + \Delta\mathbf{T}_{min}^{t,*} \right)\right\}\right\| + \left\| min\left\{0, \left(\bar{\mathbf{T}}^{t} - \mathbf{T}^{t}(t) + \Delta\mathbf{T}_{max}^{t,*} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

$$x(t)\big|_{k+R} = 0 \qquad t=k+R$$

FORMULATION OF NMPC WITH Piecewise-Continuous Control-Vector Parameterization

$$\underset{\mathbf{u}(i),\ldots,\mathbf{u}(i+m-1)}{\textit{minimize}} \quad \Phi\left(\mathbf{u}(i)\right) = \int_{t_i}^{t_{k+R}} \left\| \mathbf{T}_k^{t,opt}(\infty) - \mathbf{T}_{MP}^{t}(i) \right\| dt \qquad i=k,\ldots,k+R$$

$$\mathbf{T}_{MP}^{t}(i) = \mathbf{T}_{M}^{t}(i) + \mathbf{d}(k) \qquad i=k,\ldots,k+R$$

$$\mathbf{d}(k) = \mathbf{T}_{P}^{t}(k) - \mathbf{T}_{M}^{t}(k)$$

$$\dot{\mathbf{x}}(t,z) = \boldsymbol{f}\left(\mathbf{x}(t,z), \partial\mathbf{x}(t,z)/\partial z, \mathbf{u}(t), \mathbf{p}, \mathbf{d}\right) \qquad t \in [k,\ldots,k+R]$$

$$\mathbf{x}(k,z) = \bar{\mathbf{x}}(z)$$

$$\Delta\mathbf{T}_{min}^{t,*} \le \left\{ \mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} \right\} \le \Delta\mathbf{T}_{max}^{t,*} \qquad t \in [k,\ldots,k+R]$$

$$(1-r)\bar{\mathbf{u}} \le \mathbf{u}(t) \le (1+r)\bar{\mathbf{u}} \qquad t \in [k,\ldots,k+R]$$

$$\dot{x}(t) = \left\| min\left\{0, \left(\mathbf{T}^{t}(t) - \bar{\mathbf{T}}^{t} + \Delta\mathbf{T}_{min}^{t,*} \right)\right\}\right\| + \left\| min\left\{0, \left(\bar{\mathbf{T}}^{t} - \mathbf{T}^{t}(t) + \Delta\mathbf{T}_{max}^{t,*} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

$$x(t)\big|_{k+R} = 0 \qquad t=k+R$$

$$\dot{y}(t) = \left\| max\left\{0, \left(\left| \mathbf{u}(t) - \bar{\mathbf{u}} \right| - r\bar{\mathbf{u}} \right)\right\}\right\| \qquad t \in [k,\ldots,k+R]$$

$$y(t)\big|_{k+R} = 0 \qquad t=k+R$$

for SPCCVLP:

$$\mathbf{u}(t) = \boldsymbol{P}_i(t) \qquad t \in [t_i,\ldots,t_{i+1}]$$

$$\boldsymbol{P}_i(t) = \boldsymbol{P}_{i-1}\left(t^{-}\right) \qquad t^{-} \in [t_{i-1},\ldots,t_i]$$

$$\frac{d\boldsymbol{P}_i(t)}{dt} = \frac{d\boldsymbol{P}_{i-1}\left(t^{-}\right)}{dt} \qquad t^{-} \in [t_{i-1},\ldots,t_i]$$

for SPCCVGP:

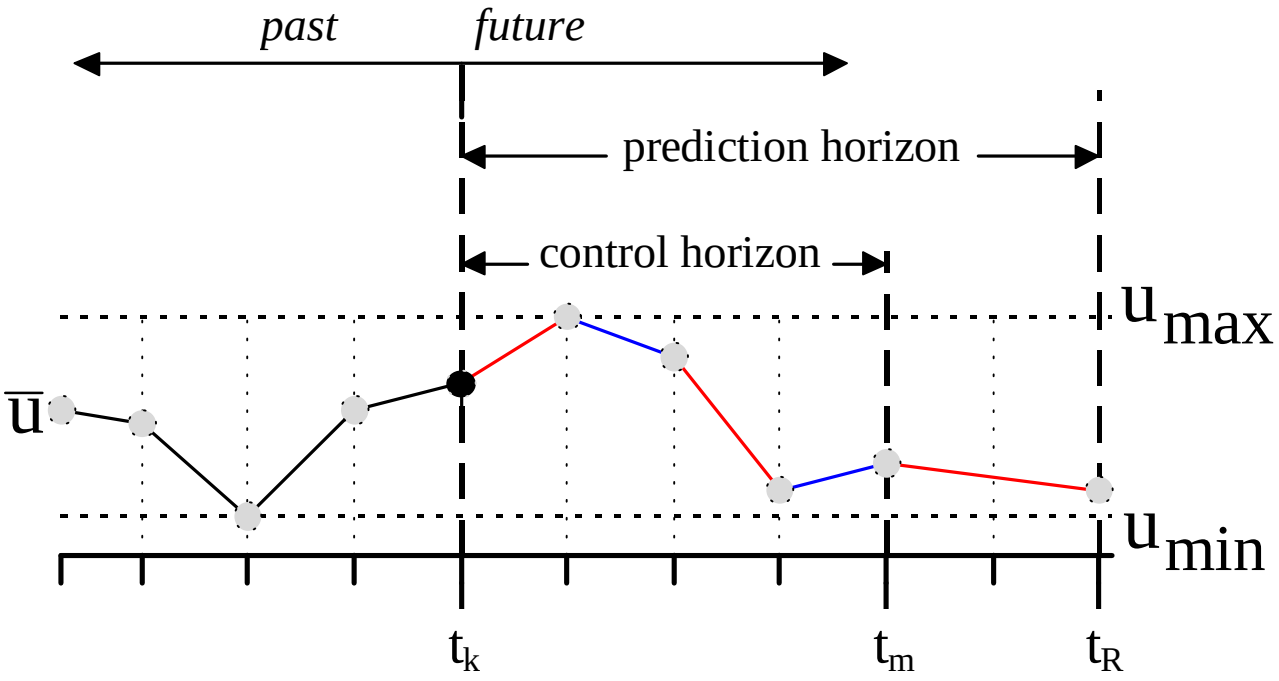
$$\mathbf{u}(t) = \boldsymbol{P}_k(t) \qquad t \in [t_k,\ldots,t_{k+R}]$$

$$\boldsymbol{P}_k(t) = \boldsymbol{P}_{k-1}\left(t^{-}\right) \qquad t^{-} \in [t_{k-1},\ldots,t_{k-1+R}]$$

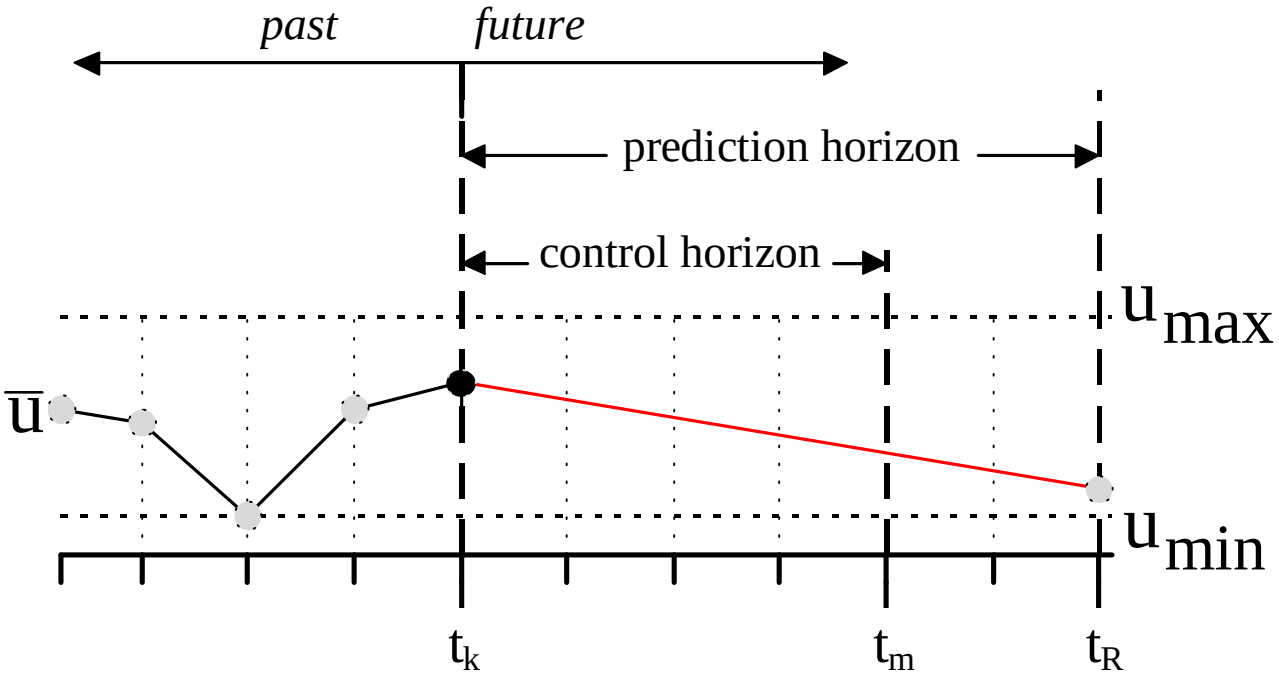
$$\frac{d\boldsymbol{P}_k(t)}{dt} = \frac{d\boldsymbol{P}_{k-1}\left(t^{-}\right)}{dt} \qquad t^{-} \in [t_{k-1},\ldots,t_{k-1+R}]$$

NMPC with Piecewise-Continuous Control-Vector Parameterization

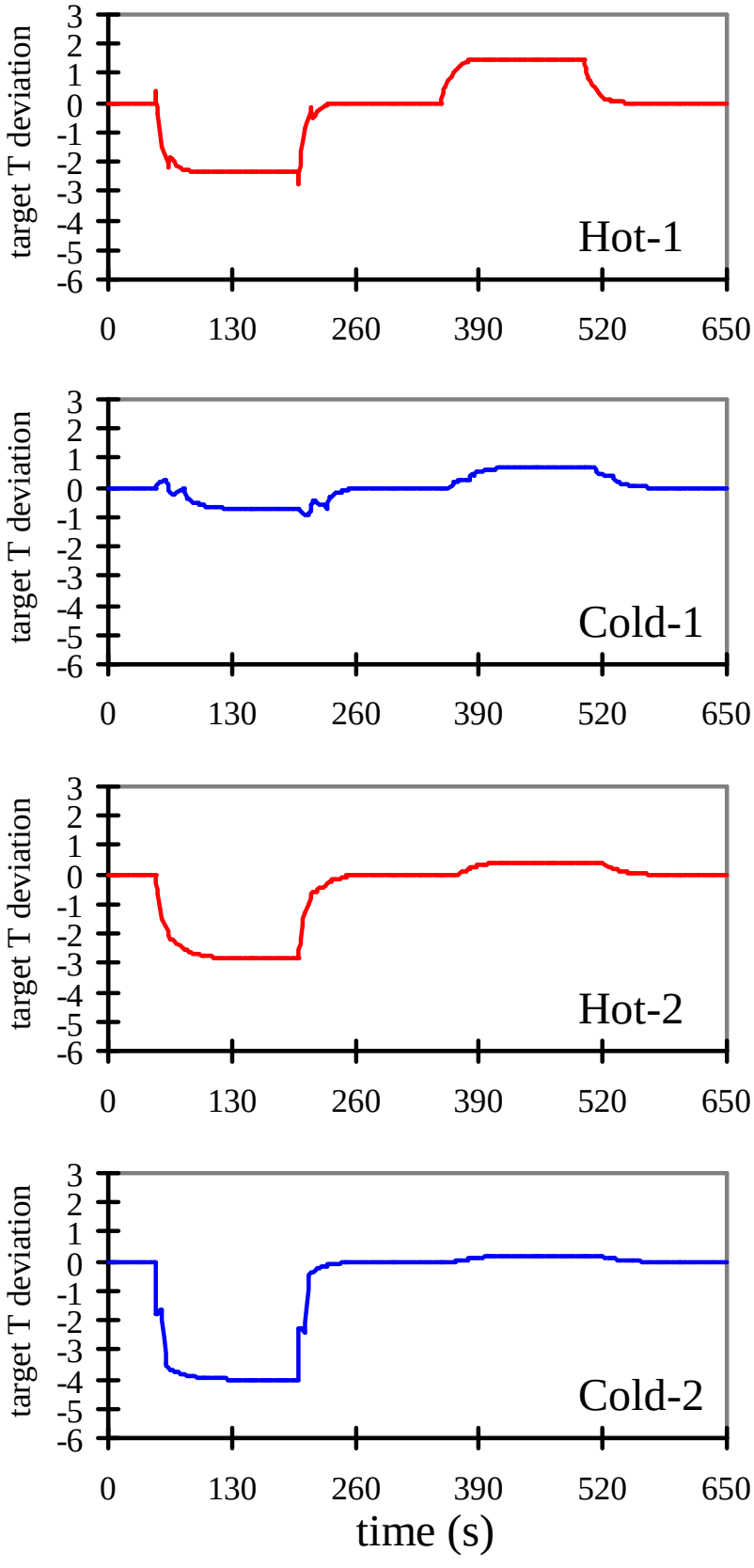
SPCCVLP: Local Parameterization (linear)



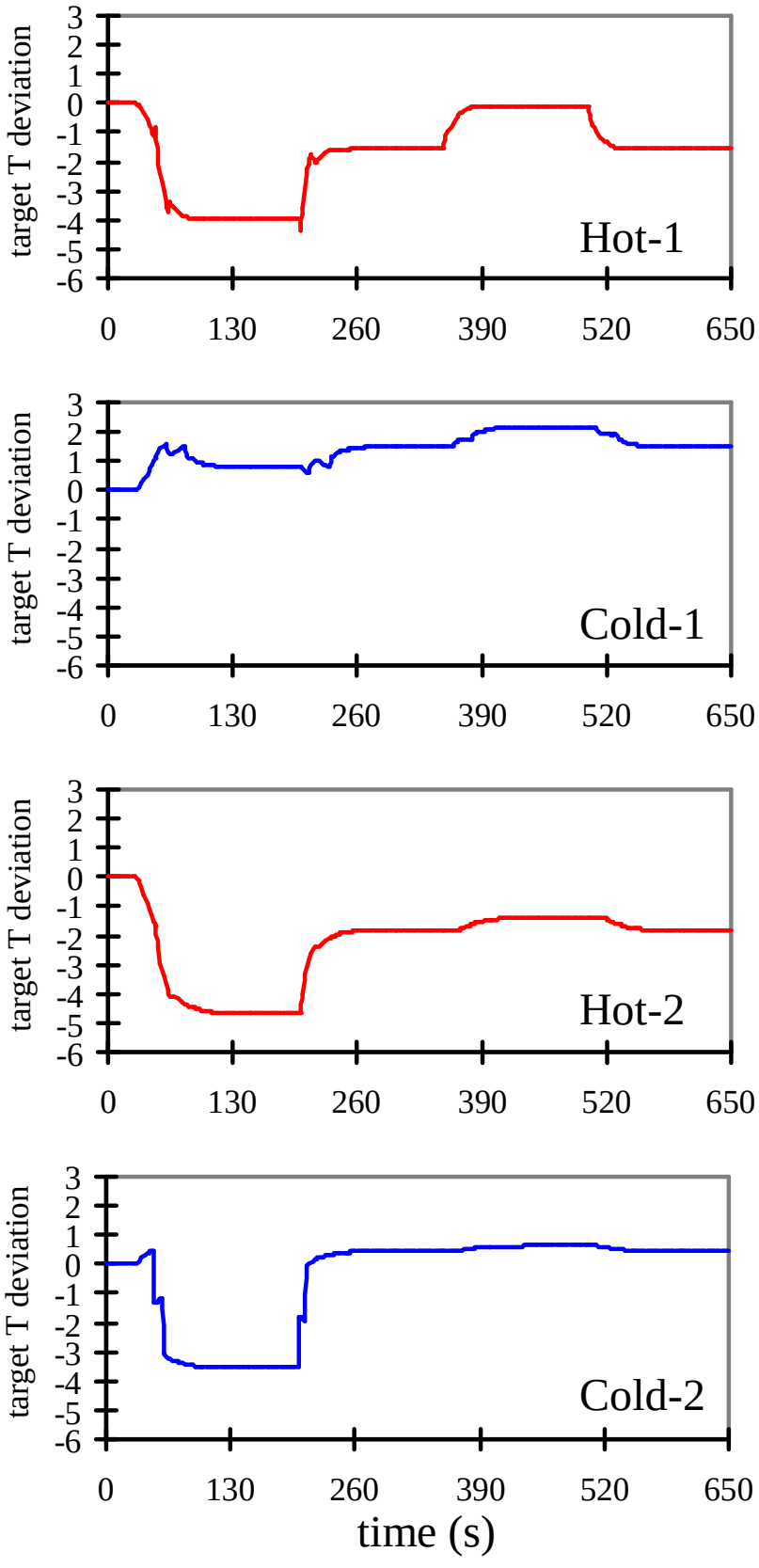
SPCCVGP: Global Parameterization (linear)



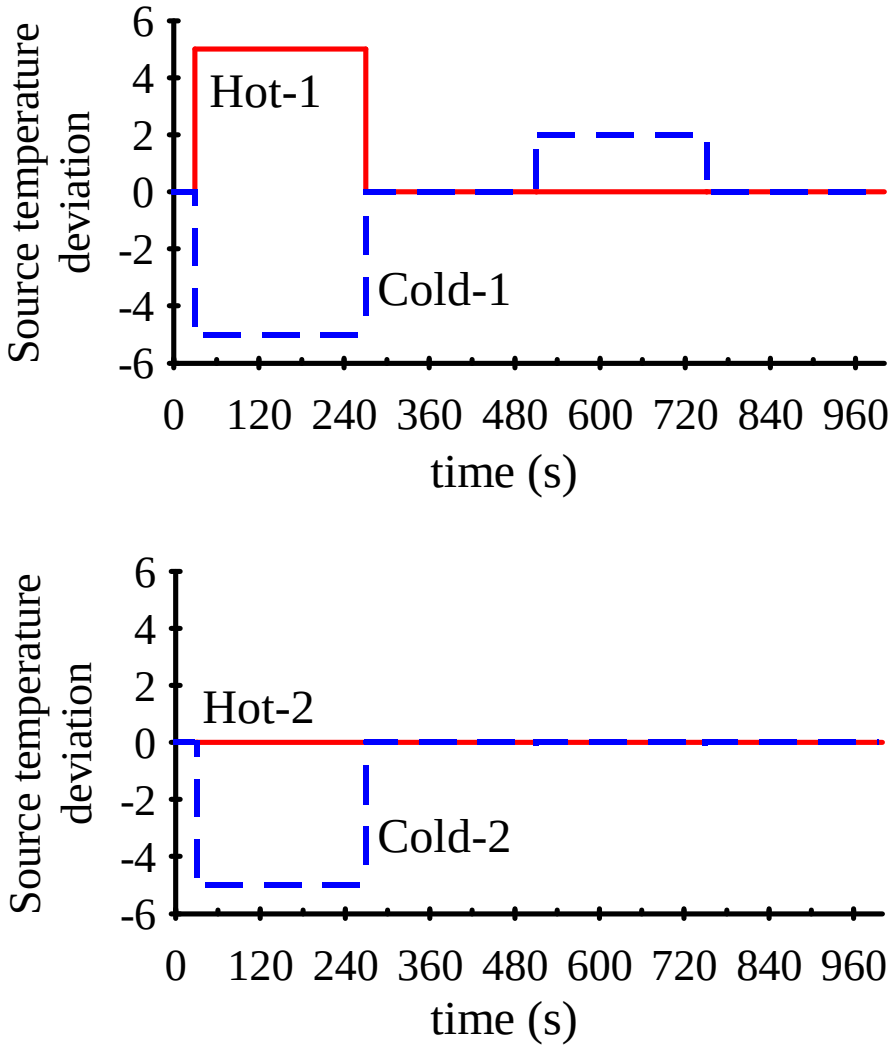
Uncontrolled Plant Response without Plant/Model Mismatch Error



Uncontrolled Plant Response with Plant/Model Mismatch Error



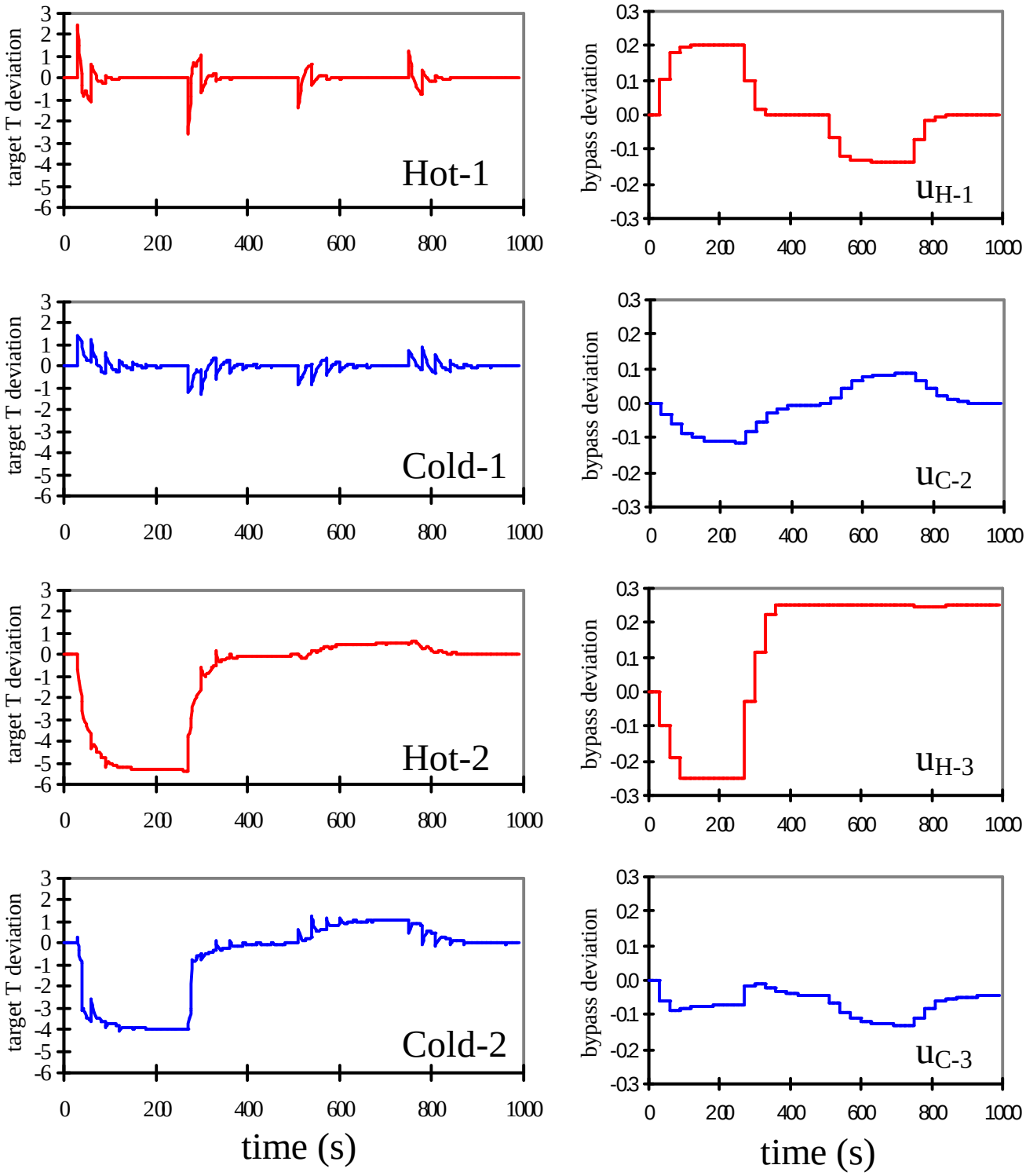
Consecutive Disturbances Introduced in the Source-Stream Temperatures



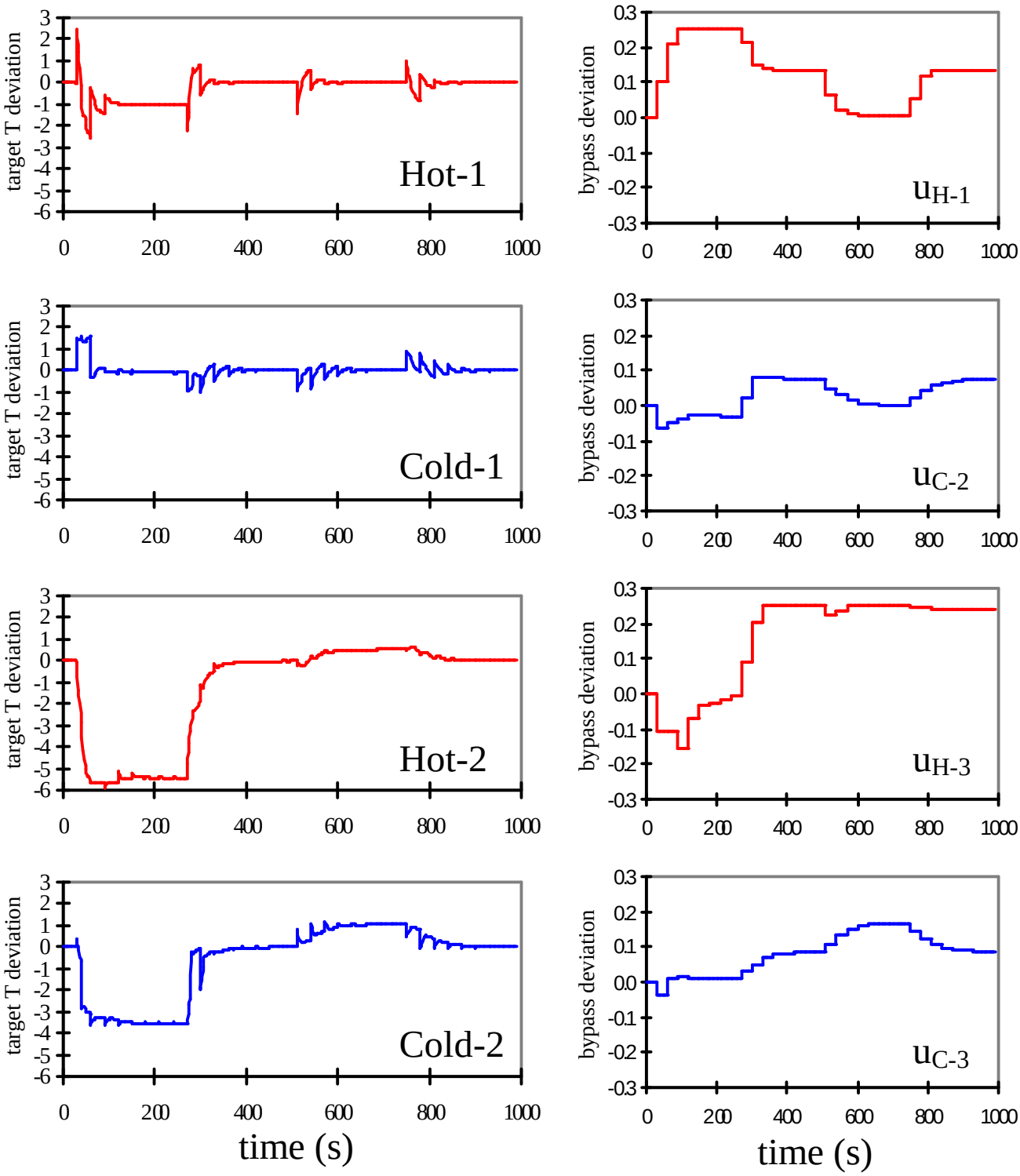
Some parameters used in NMPC schemes

$\Delta t = t_{k+1} - t_k = 30$ s	m = 3 steps	R = 4 steps
$t_R - t_m = \Delta t$	r = 0.25	r* = 0.1

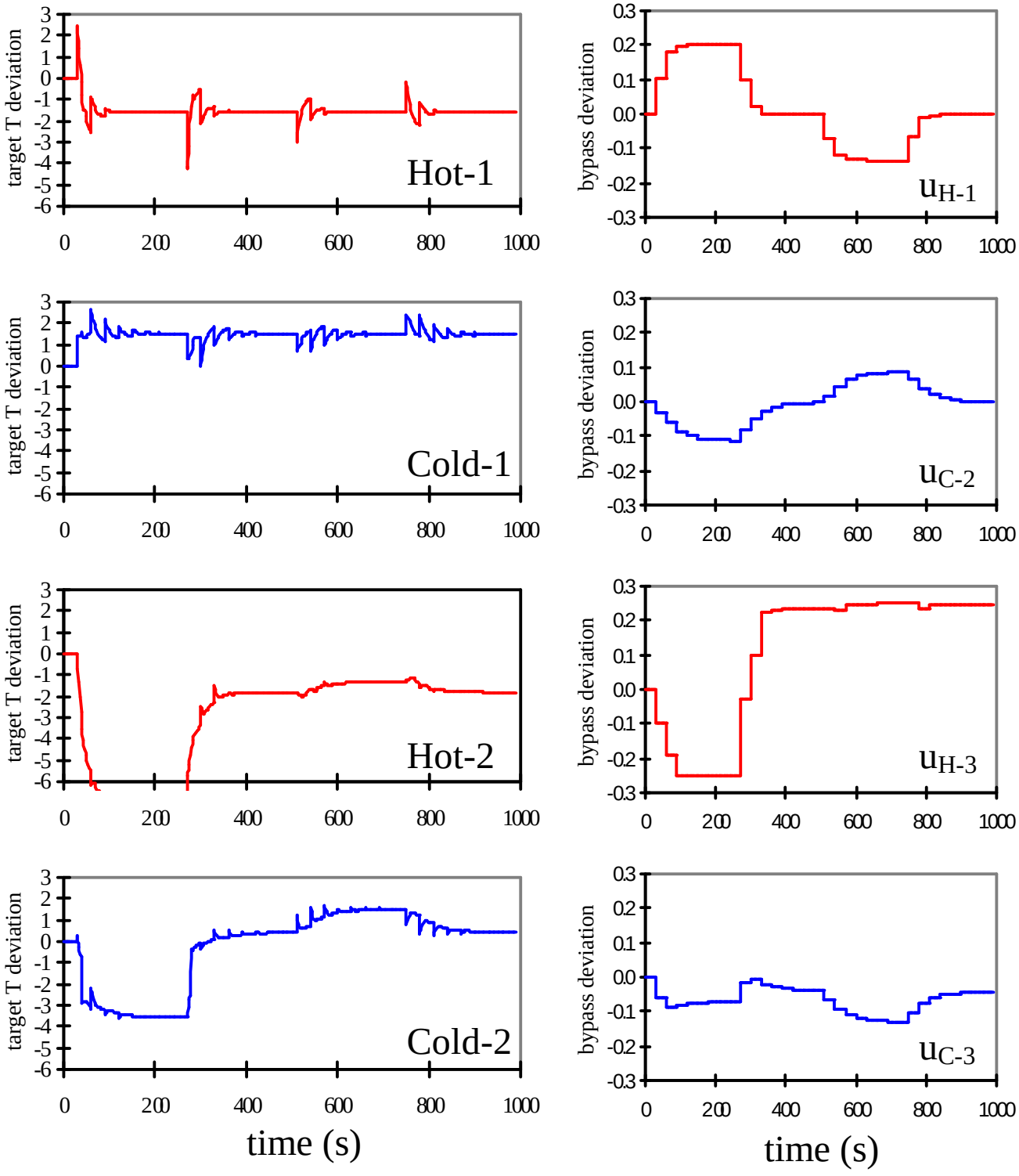
NMPC
without Plant/Model Mismatch



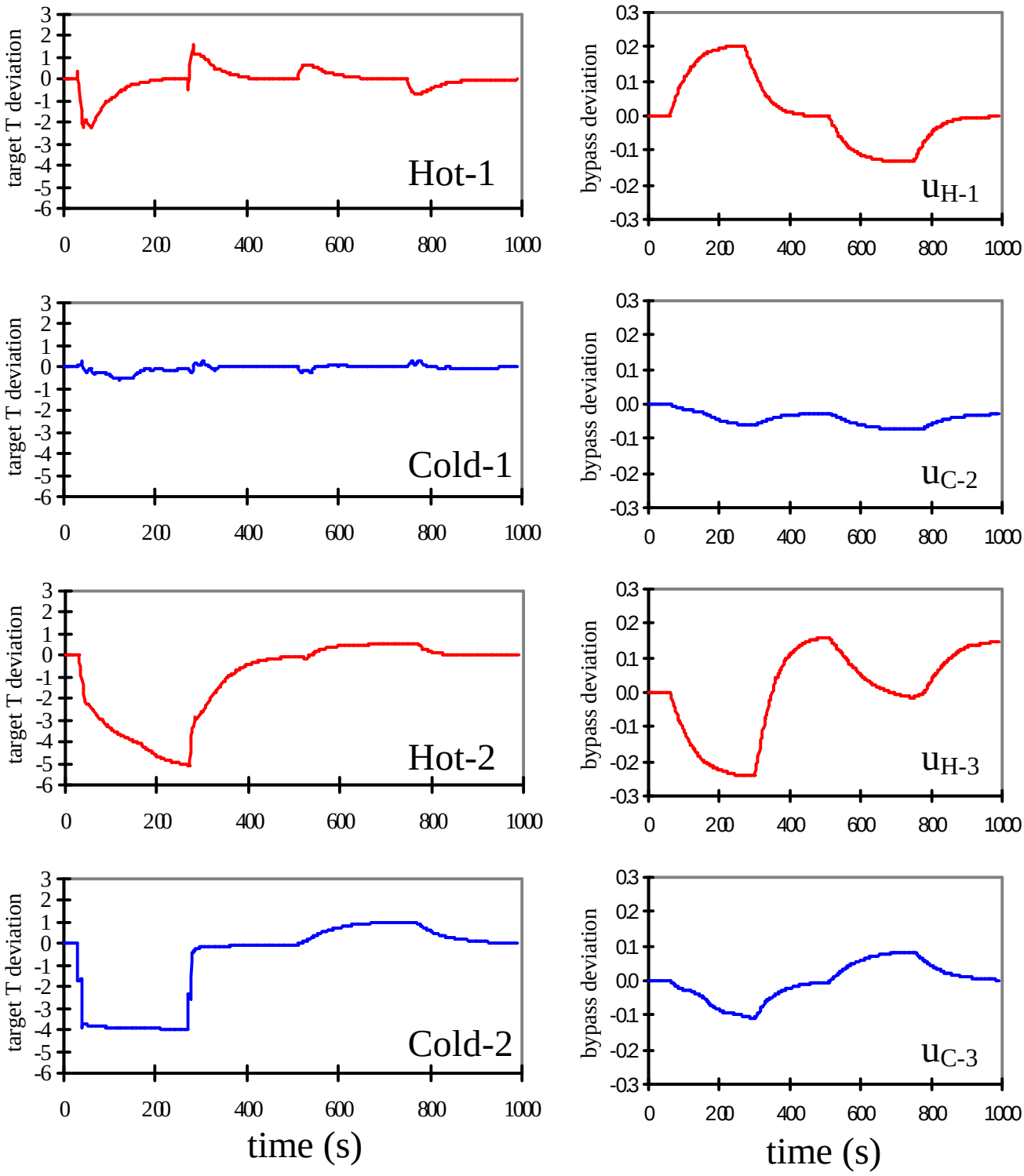
NMPC
with Plant/Model Mismatch



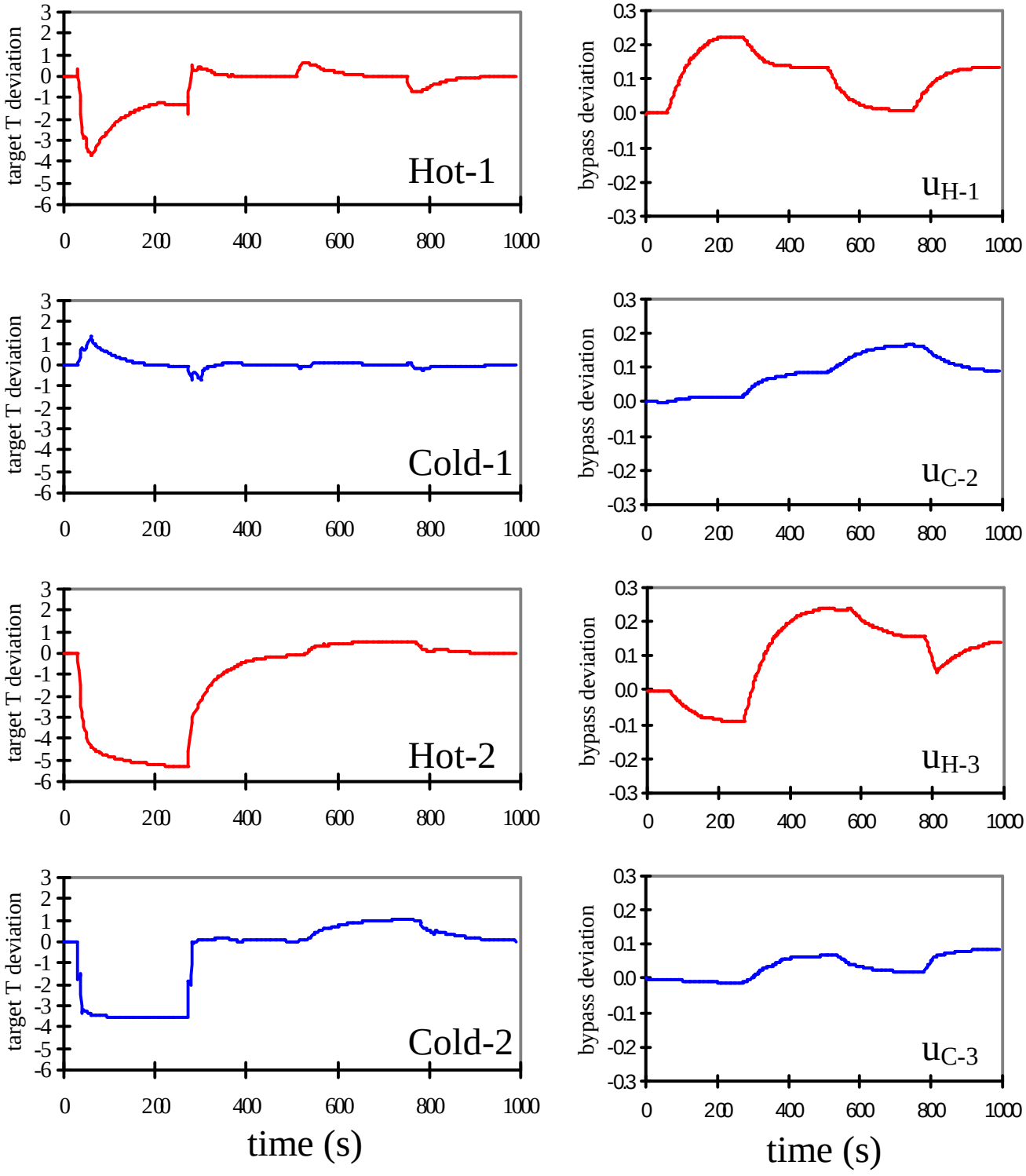
NMPC
with Plant/Model Mismatch
but *d* is not taken into account



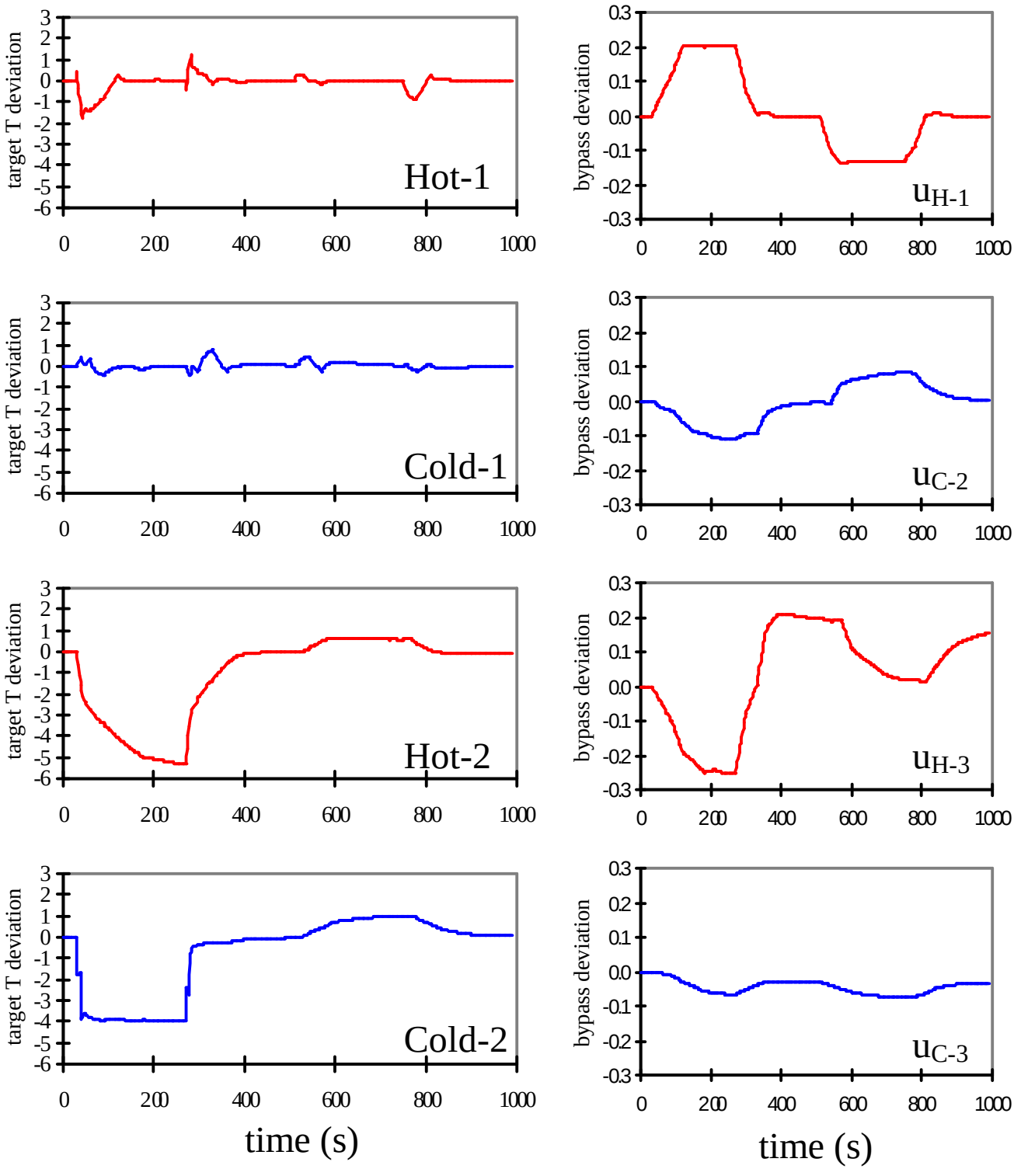
NMPC / SPCCVGP
global & linear parameterization
without Plant/Model Mismatch



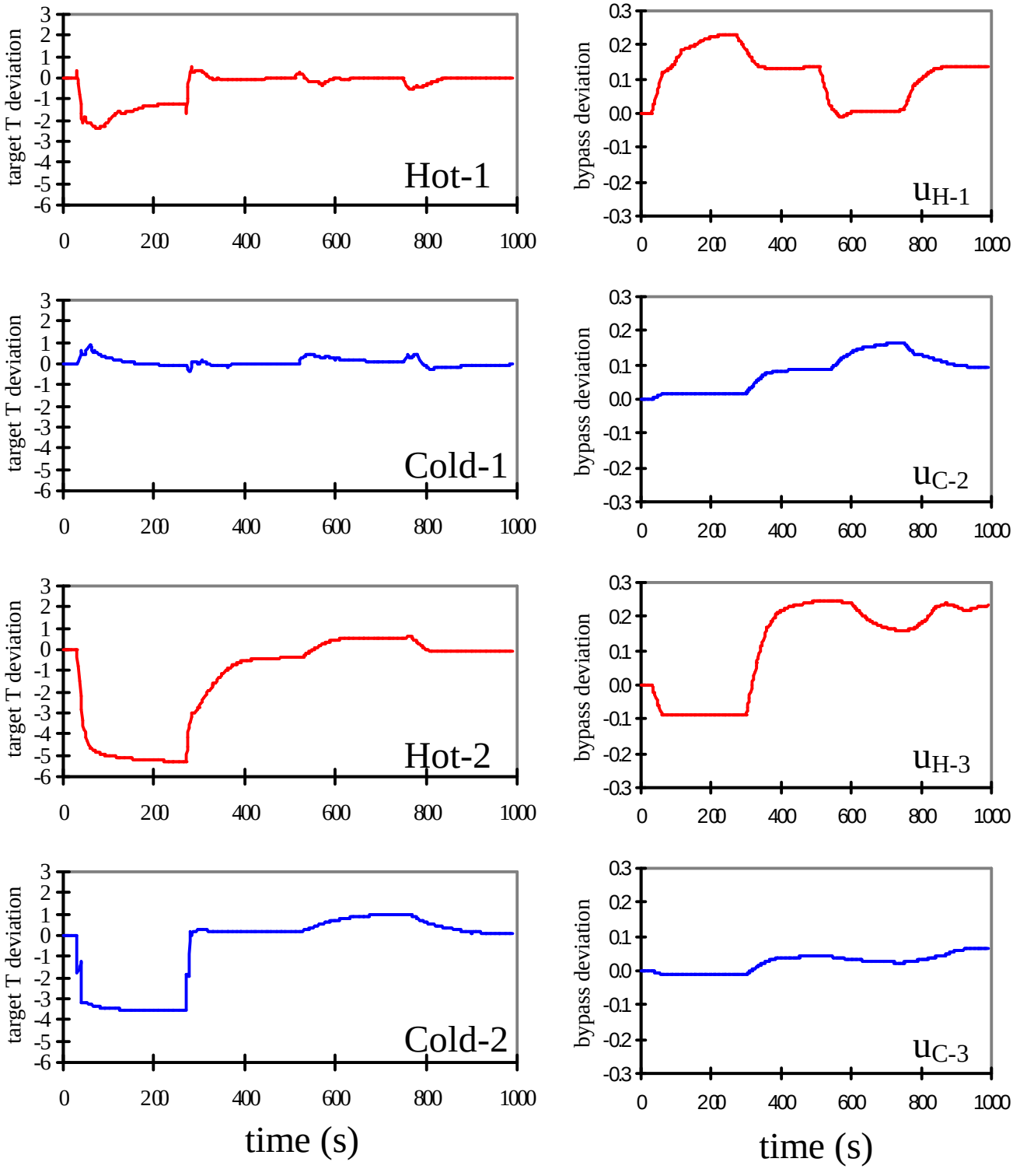
NMPC / SPCCVGP
global & linear parameterization
with Plant/Model Mismatch



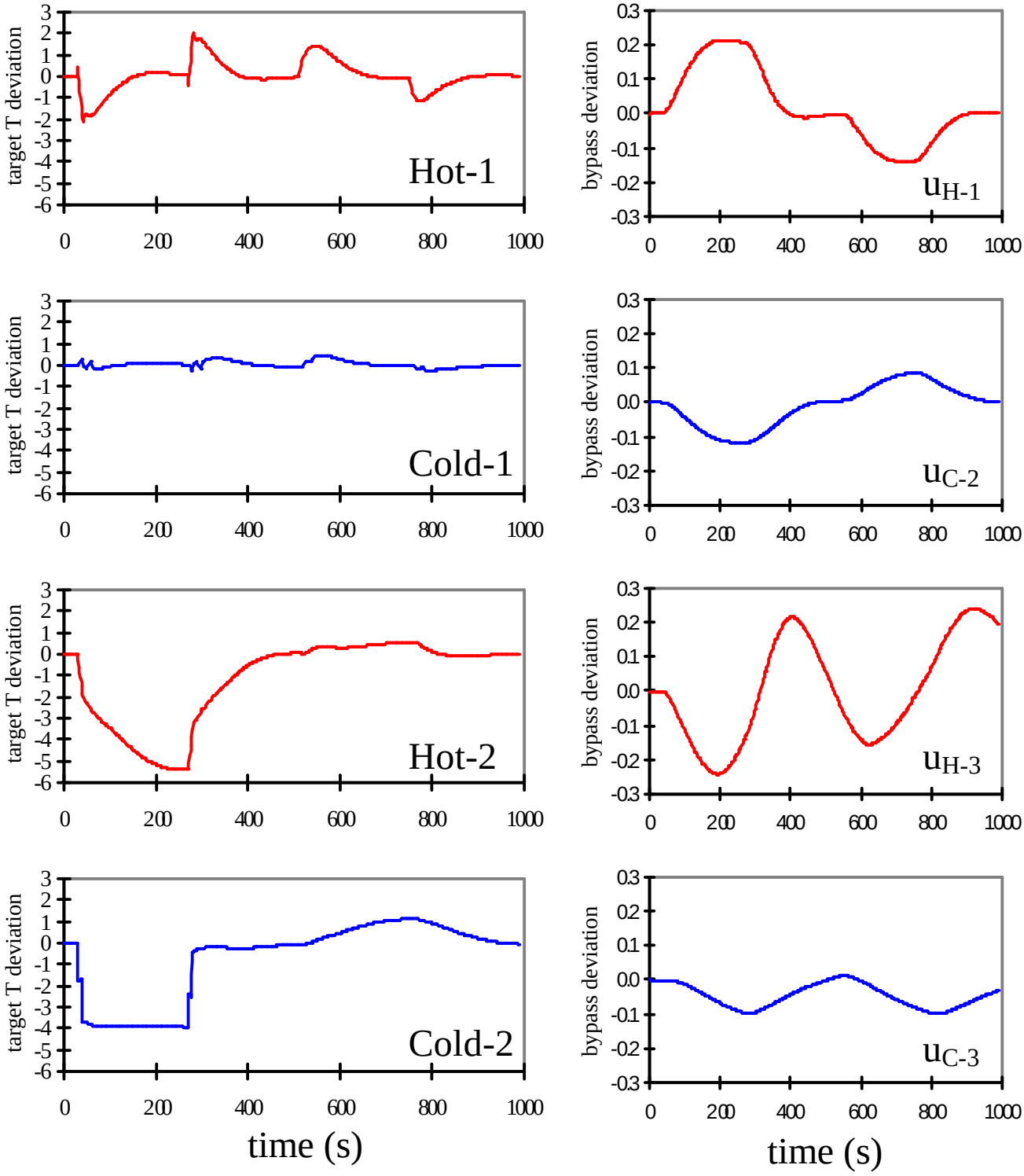
NMPC / SPCCVLP
local & linear parameterization
without Plant/Model Mismatch



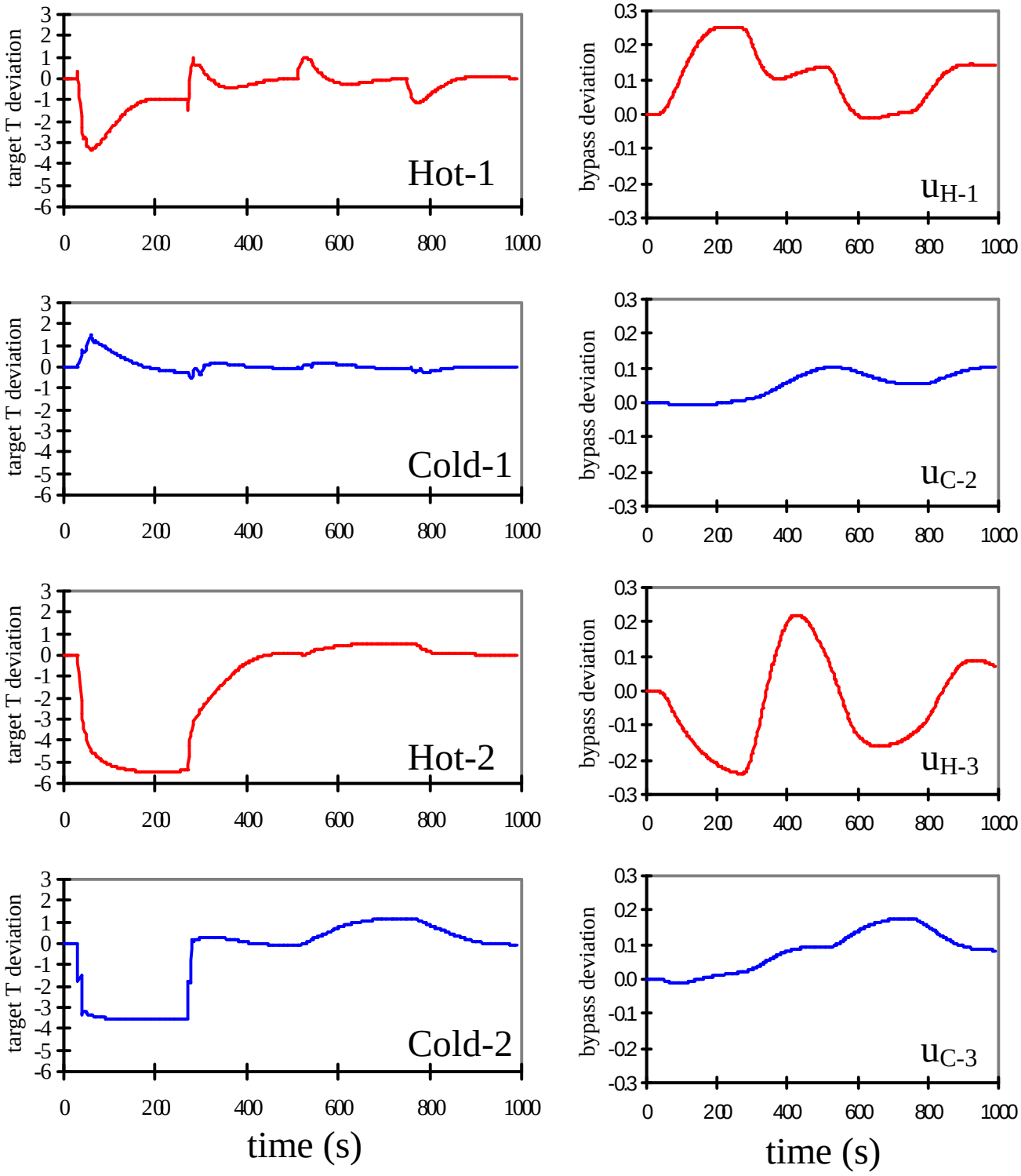
NMPC / SPCCVLP
local & linear parameterization
with Plant/Model Mismatch



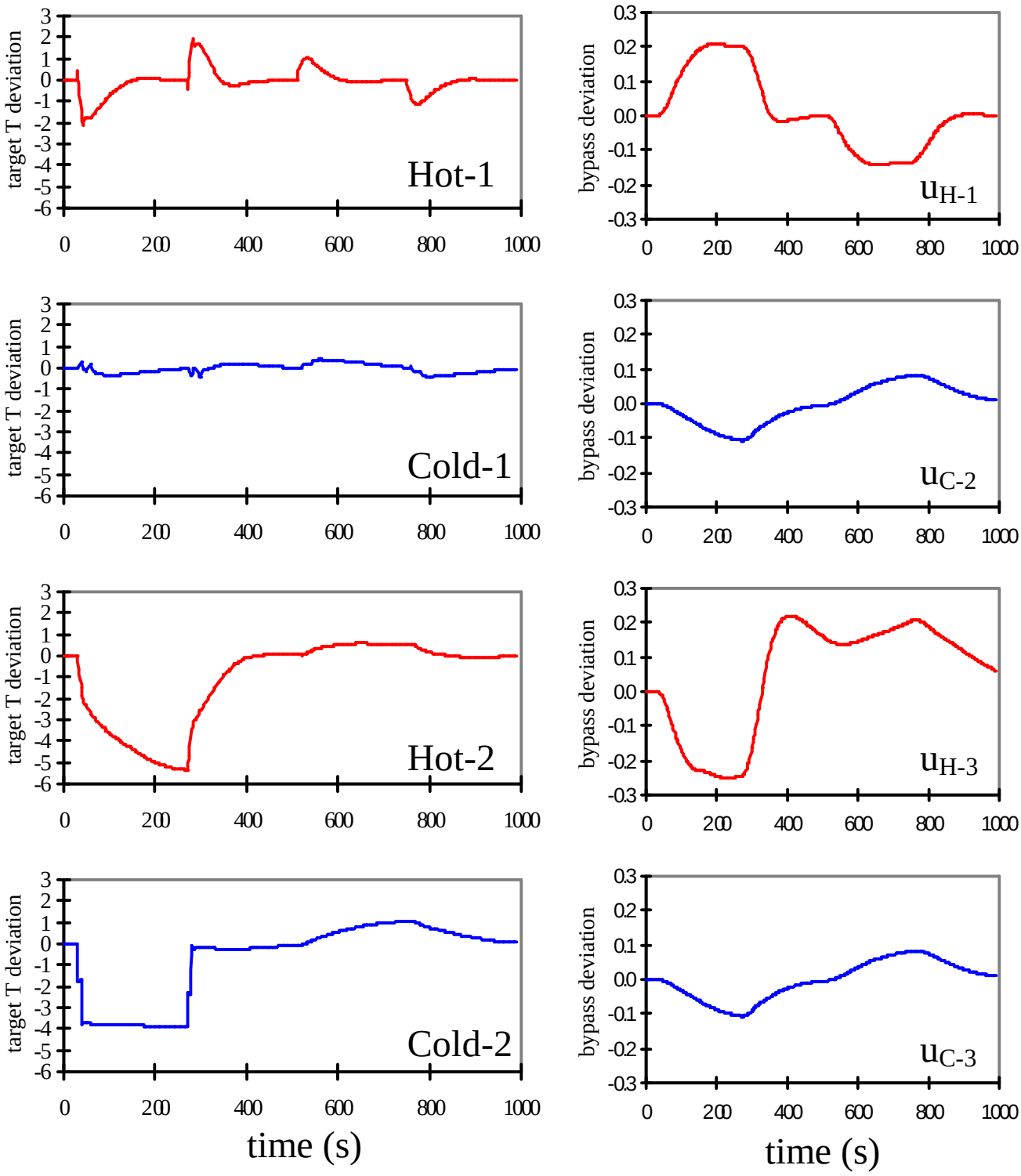
NMPC / SPCCVGP
global & quadratic parameterization
without Plant/Model Mismatch



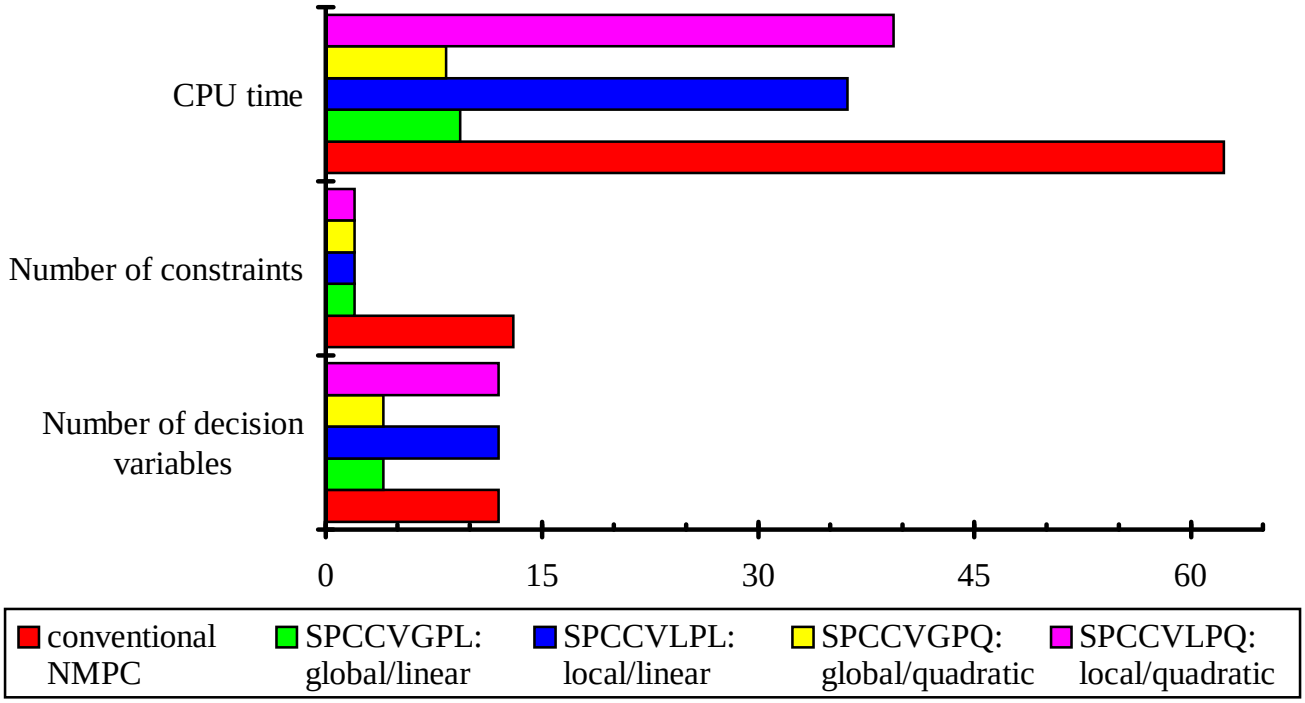
NMPC / SPCCVGP
global & quadratic parameterization
with Plant/Model Mismatch



NMPC / SPCCVLP
local & quadratic parameterization
without Plant/Model Mismatch



Comparison of NMPC with SPCCVP`s



CONCLUSIONS

- Retrofit design of a HEN was performed considering placement of manipulated variables (bypasses) in order to increase HEN`s flexibility and resiliency.
- NMPC of the retrofit HEN modeled with PDEs was studied.
- A steady-state optimization algorithm, which determines feasible (optimal) post-disturbance target temperatures, was combined with the NMPC algorithm.
- The conventional NMPC scheme satisfied all the temporal and steady-state hard/soft target-temperature constraints even in the presence of a significant plant/model mismatch, however with nonsmooth response.
- A version of the classical NMPC scheme, NMPC/SPCCVGP, which has a piecewise-continuous linear-global parameterization of the control-vector, showed a superior performance with regard to smoothness of the response and significant reduction in computational load.
- Quadratic-global parameterization performed slightly better than the linear-global parameterization in terms of the smoothness of the response at no additional computational load.
- Local parameterizations (linear or quadratic) also gave smoother responses, however with no benefit with respect to computational load compared to the load in the conventional NMPC.

ACKNOWLEDGMENT

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