Nonlinear Model-Predictive Control with Input Parameterization: A Comparative Study on the Van de Vusse Reactor

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Abstract – A Nonlinear Model-Predictive Control (NMPC) algorithm with Piecewise-Continuous Control-Vector Parameterization (PCCVP) (input parameterization through basis functions) is presented and tested on the Van de Vusse reactor. The primary aim for the use of input-parameterization in NMPC is to decrease the high computation-time requirements of the dynamic optimization that involves a nonlinear dynamic model constrained with nonlinear equality and inequality constraints. The secondary aim is to achieve smooth system responses through the use of piecewise-continuous input profiles. It is demonstrated that the various NMPC-PCCVP schemes suggested produce smoother and more effective control with respect to the conventional NMPC. The NMPC-PCCVP schemes enable the completion of dynamic-optimization tasks within significantly reduced time intervals.

Introduction

Following the successful industrial applications of linear model-based control algorithms, the NMPC algorithms are getting increasingly more attention for the control of nonlinear systems (Bequette, 1991). The NMPC solves a real-time nonlinear dynamic-optimization problem based on predicted future response of a nonlinear system. A set of future controls (manipulated-variable moves) is optimized such that an objective function (deviations in controlled variables from optimal post-disturbance values over prediction horizon) is minimized subject to general nonlinear equality and/or inequality path or point constraints.

For large-scale NMPC applications, the main difficulty is due to the size of the nonlinear dynamic-optimization problem that has to be solved on-line with the process. A large-scale and computationally expensive general dynamic-optimization problem has to be solved within the time interval between two control moves as they are actually applied to plant. For a plausible length of prediction with several manipulated variables, the number of decision variables can easily go beyond the point of computational feasibility, and hence, the general nonlinear constrained dynamic-optimization problem cannot be solved within restricted sampling/implementation intervals. Since the computation time increases exponentially with respect to the number of controlled variables, the situation worsens for large-scale systems.

There are few methods proposed to overcome the lack of computer power faced in the solution of large-scale nonlinear dynamic-optimization problems (Zheng, 1997). Garcia (1986) proposed linearization of the model. Zheng (1997) suggested calculating the first move only and approximating the never-implemented decision variables. Ricker (1985) proposed the blocking of manipulated-variables to decrease the computational effort. In manipulated-variable blocking, some of the manipulated variables are held constant, hence dropped off the

optimization problem for the sake of decreasing the computation time; at the expense of decreasing the effectiveness of control.

Akman (1997) suggested a PCCVP approach for computationally efficient implementation of the general NMPC. The piecewise-continuous parameterization of the controls, as opposed to the use of piecewise-constant (discrete) controls in the conventional MPC, smooths out the objective function and thus demands fewer number of optimization iterations. This technique is identical to the conventional NMPC if zero-order parameterization is used. Akman (1997) applied both the local and global forms of control-vector parameterization for the control of a Heat-Exchanger Network (HEN). This sample HEN system possesses the capability of implementing a continuous input-profile and therefore the parameterized input profile can be implemented to the system in a continuous fashion. Akman (1997) observed that the NMPC-PCCVP scheme produced a smoother input profile, and hence a smoother output profile that yielded a more acceptable control performance. Additionally, the NMPC-PCCVP was demonstrated to decrease the computation-time with respect to the conventional NMPC even when the number of decision variables was equal.

Nonlinear-Model Predictive Control

The term NMPC describes a class of computer-control algorithms that control the future behavior of a plant through the use of an explicit nonlinear process model. At each control interval, the NMPC algorithm computes an open-loop sequence of manipulated-variable adjustments to optimize the future plant behavior. The first input in the optimal sequence is implemented on the plant and the entire optimization is repeated at subsequent control intervals. The conventional NMPC scheme is depicted in Fig.1.

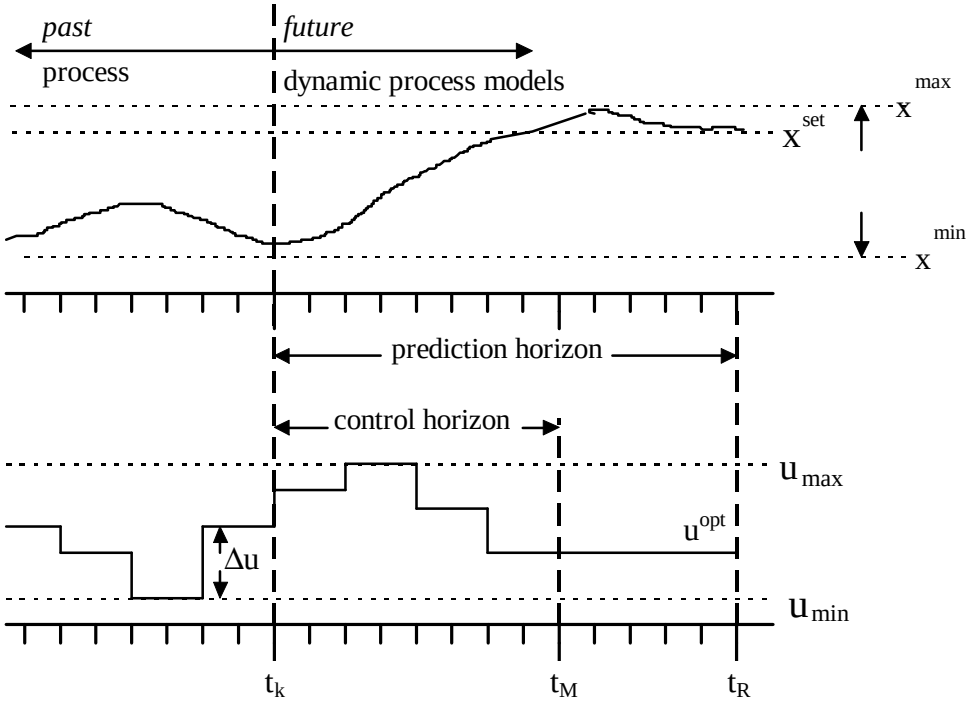


Figure 1. Classical NMPC scheme.

The classical NMPC problem can be formulated as a constrained dynamic-optimization problem:

$$\underset{\mathbf{u}(k+m)}{\text{minimize}}\ \Phi\left(\mathbf{u}(k+m)\right)=\int_{t_k}^{t_{k+R}}\left(\mathbf{x}^{MP}(t)-\mathbf{x}^{set}\right)^T\mathbf{Q}\left(\mathbf{x}^{MP}(t)-\mathbf{x}^{set}\right)dt \qquad m=1,\ldots,M \qquad (1)$$

$$\mathbf{x}^{MP}=\mathbf{x}^{P}+\mathbf{d}_k^{p/m} \qquad (2)$$

$$\mathbf{d}_k^{p/m}=\mathbf{x}_k^{P}-\mathbf{x}_k^{M} \qquad (3)$$

$$\frac{d\mathbf{x}}{dt}=\mathbf{f}\left(\mathbf{x}(t),\mathbf{u}(t),\mathbf{d}(t),t\right) \qquad (4)$$

$$\mathbf{u}_{k+j}(t)=\mathbf{u}_{k+M}(t) \qquad \forall j>M \qquad (5)$$

$$\mathbf{u}^{min}\le\mathbf{u}_{k+j}(t)\le\mathbf{u}^{max} \qquad j=1,\ldots,R \quad t\in[t_k,t_{k+R}] \qquad (6)$$

$$\Delta\mathbf{u}^{min}\le\left(\mathbf{u}_{k+j+1}(t)-\mathbf{u}_{k+j}(t)\right)\le\Delta\mathbf{u}^{max} \qquad j=1,\ldots,R \quad t\in[t_k,t_{k+R}] \qquad (7)$$

$$\mathbf{x}^{min}\le\mathbf{x}(t)\le\mathbf{x}^{max} \qquad t\in[t_k,t_{k+R}] \qquad (8)$$

$$\mathbf{h}\left(\mathbf{x}(t),\mathbf{u}(t),\ t\right)=\mathbf{0} \qquad t\in[t_k,t_{k+R}] \qquad (9)$$

$$\mathbf{g}\left(\mathbf{x}(t),\ \mathbf{u}(t),\ t\right)\le\mathbf{0} \qquad t\in[t_k,t_{k+R}] \qquad (10)$$

In the formulation given above, M is the control horizon and R is the prediction horizon. The objective function given in Eq.1 is the square of the norm of the residuals between the outputs (observed variables) and the set points, integrated over the prediction horizon. The weights of the individual states are determined by the elements of matrix **Q**, which is a diagonal matrix so that only the quadratic terms in the matrix product given in Eq.1 are considered in the objective function. Decision variables are the control actions considered only M time steps into the future, and after the M^{th} step, control action is kept constant by Eq.5. The system is defined by the vector of nonlinear differential equations given in Eq.4. Compensation for the plant/model mismatch is considered in Eqs.2 and 3 where the difference between the model output and the plant output at the current step k ($\mathbf{d}_k^{p/m}$) is used to predict the output over the prediction horizon. At the current time, k^{th} step, the objective, the integral square error, is minimized over the prediction horizon, and the first control action is implemented to the system, and the calculations are repeated in the next step. The absolute bounds on the manipulated variables are given in Eq.6, and the absolute bounds on the state variables are given in Eq.8. Velocity constraints limiting the movement rate of the manipulated variables are defined by Eq.7.

Eqs.9 and 10 are the general equality- and inequality-constraint vectors. The state, manipulated-variable, and velocity constraints defined above are some of the special types of these general constraints, and are defined explicitly since they are the standard constraints for the NMPC problem. General constraint forms are written to enable the formulation of other constraints within the general formulation, such as terminal constraints or contraction constraints.

Control-Vector Parameterization

An alternative to the classical NMPC scheme, is the NMPC-PCCVP (Akman, 1997). In the NMPC-PCCVP, the manipulated variables (inputs) are expressed in a predefined form, in the form of a basis-function, such as polynomial splines. Control performance is optimized with respect to the parameters in the basis function. This method enables defining a complex input profile over a potentially infinite control horizon, using a small number of unknown parameters. The decision on the type of the basis function used establishes many features of the computed input profile, and the success of the NMPC-PCCVP is significantly dependent upon the appropriateness of the predefined form of the input. Various parameterization techniques can be adopted. An alternative is the local-parameterization. In this case, the input profile is expressed as a vector of consecutive piecewise-continuous splines; each spline corresponding to a step in the conventional NMPC. It is possible to use linear, quadratic or higher-order polynomial splines. However, as the order increases the number of decision variables in the dynamic-optimization problem increases. A generalized polynomial basis function is displayed in Eq.11.

$$\mathbf{u}_{k+m}(t) = \mathbf{c}_0(k+m) + \mathbf{c}_1(k+m)\tau + \mathbf{c}_2(k+m)\tau^2 + \ldots + \mathbf{c}_H(k+m)\tau^H \quad m=1,\ldots,M \qquad (11)$$

$$\tau = t - t_{k+m-1} \qquad t \in \left[t_k, t_{k+M}\right] \qquad (12)$$

In the input-parameterization form given above, for each step in the control horizon M, there exists a separate polynomial, which is of order H, to express the manipulated-variable profile through that step. In each polynomial, there exist H+1 free parameters, $\mathbf{c}$, that are the decision variables to optimize the control. There is one polynomial corresponding to each of the manipulated variables for each step, therefore each vector $\mathbf{c}_h$ has the same dimension with the control vector **u.** Therefore, the tensor $\mathbf{c}$ has three dimensions; associated with the manipulated variable, associated with the step in the control horizon, and associated with the location in the polynomial function. The shifted time, τ, is used in the above formulation so that the parameter weights do not change in consecutive optimizations as the absolute time increases. A typical input profile in the local-linear PCCVP (LLNMPC) is depicted in Fig.2.

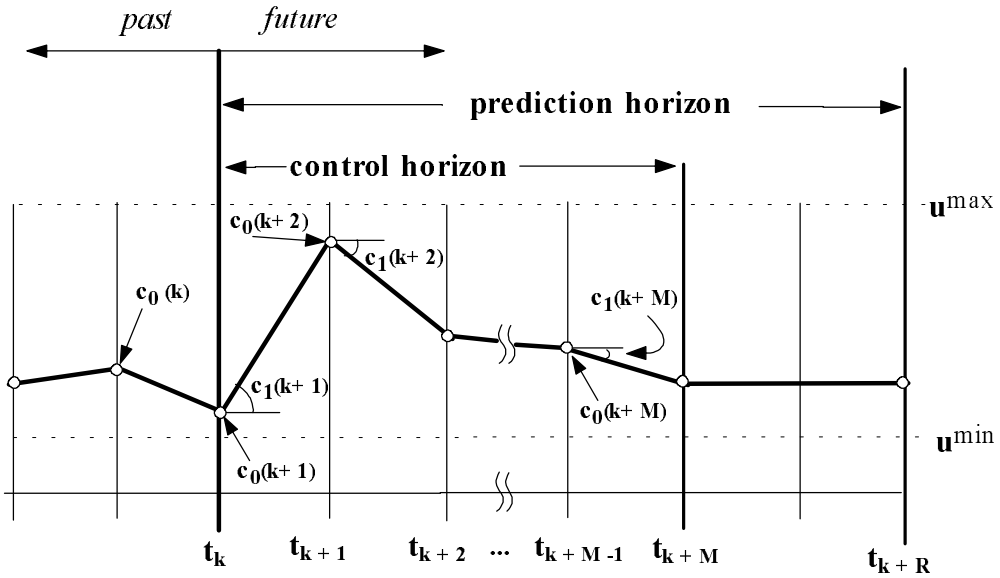


Figure 2. Sample input profile in local-linear input-parameterization.

In linear-parameterization, only the first two terms are required, as given in Eq.13, where $\mathbf{c}_0(k+m)$ is the intercept and $\mathbf{c}_1(k+m)$ is the slope of the linear-parameterization in the interval of $t \in \left[t_{k+m-1}, t_{k+m}\right]$.

$$\mathbf{u}_{k+m}(t) = \mathbf{c}_0(k+m) + \mathbf{c}_1(k+m)\,\tau \tag{13}$$

$$\tau = t - t_{k+m-1} \qquad t \in \left[t_k, t_{k+M}\right] \tag{14}$$

The conventional NMPC may be expressed as a zero-order, or piecewise-constant PCCVP of the manipulated variable, as given in Eq.15. The control profile corresponding to conventional NMPC is displayed in Fig.3.

$$\mathbf{u}_{k+m}(t) = \mathbf{c}_0(k+m) \tag{15}$$

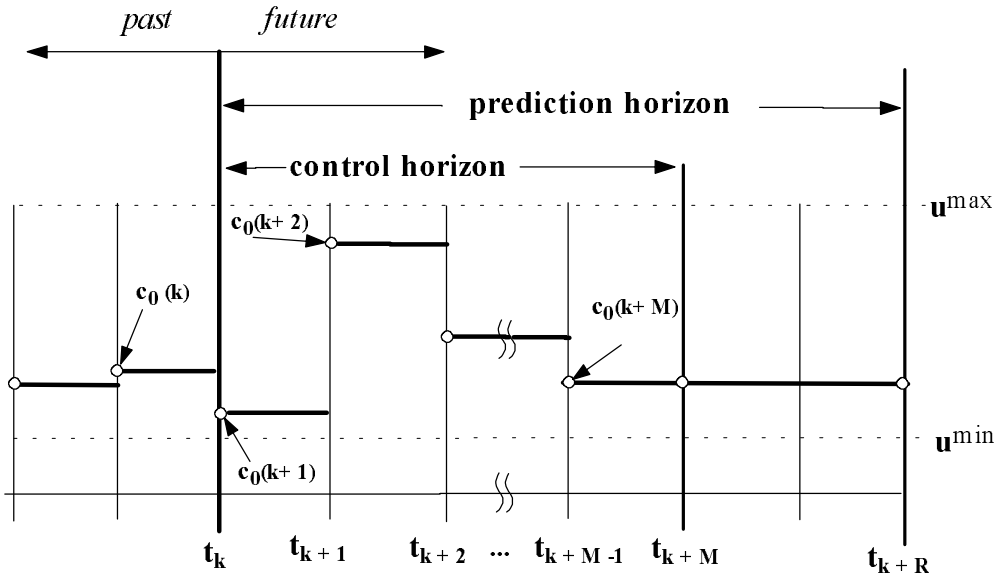


Figure 3. Sample input profile in conventional NMPC.

It is possible to parameterize the inputs globally over the entire control horizon by a single polynomial. This method keeps the number of decision variables constant; independent of the length of the control horizon. The global-linear parameterization, for instance, has only two parameters per manipulated-variable in the optimization problem, as given in Eq.16.

$$\mathbf{u}_{k+m}(t) = \mathbf{c}_0(k+1) + \mathbf{c}_1(k+1)\,\tau \tag{16}$$

$$\tau = t - t_k \qquad t \in \left[t_k, t_{k+M}\right] \tag{17}$$

The time interval counter m is dropped since a single polynomial is used throughout the entire control horizon, and accordingly the time is shifted once and at step k only, as given in Eq.17. A sample control profile under global-linear parameterization is displayed in Fig.4.

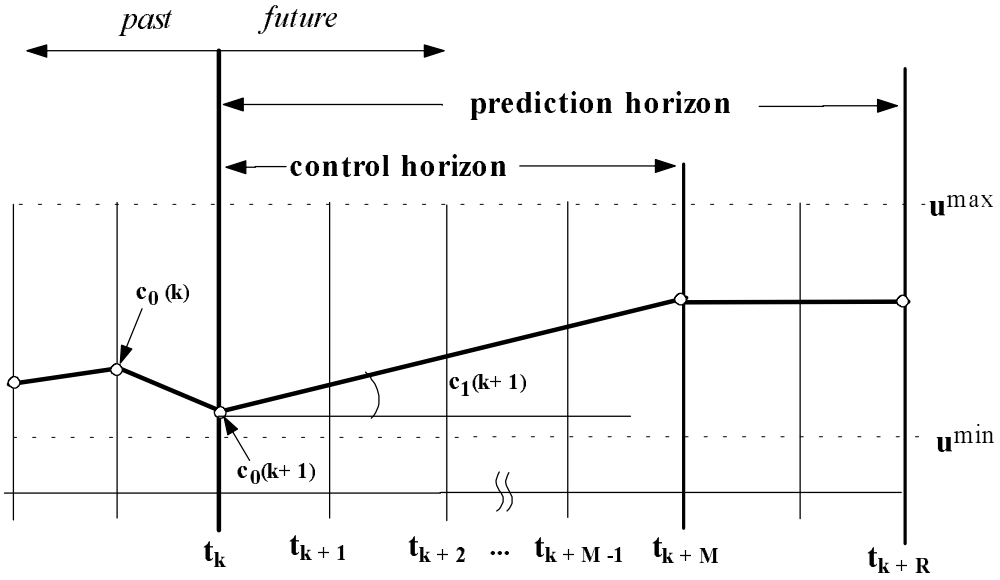


Figure 4. Sample input profile in global-linear input-parameterization.

The global parameterization suffers considerably from the limitations imposed by the predefined form. Under global-linear-parameterization, for instance, a jagged input profile as in local-linear parameterization cannot be implemented. This problem can be remedied by a higher order parameterization, however at the cost of increasing the number of undefined parameters (decision variables).

An intermediate approach between local and global input-parameterizations is the use of two consecutive splines. The first spline, parameterized over a single sampling interval, is followed by a second semi-global spline parameterized over the rest of the control horizon. A sample input profile is depicted in Fig.5.

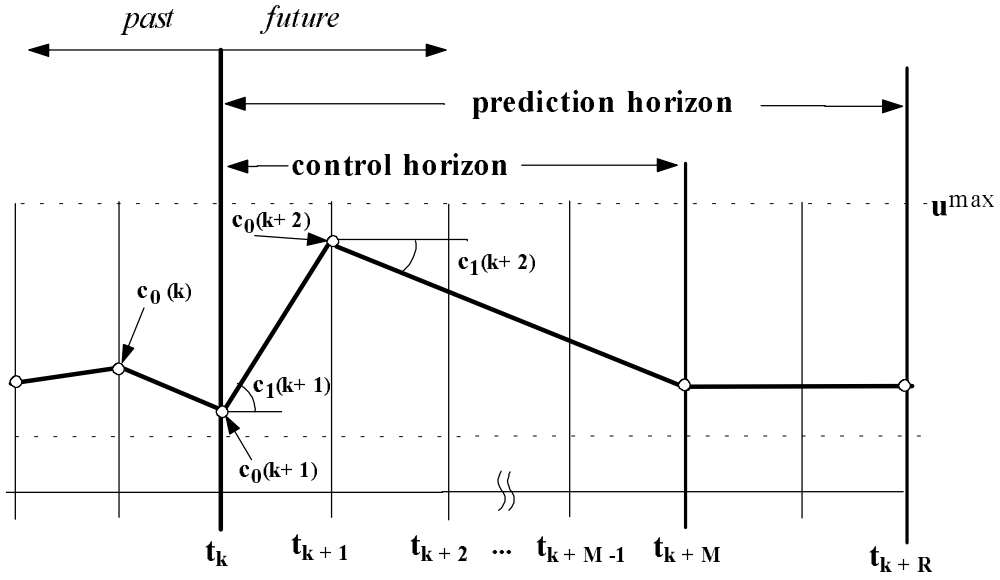


Figure 5. Sample input profile in consecutive input-parameterization.

It should be noted that in all the figures displayed above, the continuity condition is applied such that the consecutive parameterizations are interconnected to form splines. This condition decreases the required number of decision variables. Continuity is imposed by the addition of the equality constraints that require the equality of the splines at the interconnection nodes.

$$\mathbf{u}_{k+m}(t_{k+m}) = \mathbf{u}_{k+m+1}(t_{k+m}) \tag{18}$$

There exist practically infinite forms of basis functions for the PCCVP. Higher-order polynomials may be applied locally or globally, or exponential splines may be utilized. It is also possible to extend the application of the PCCVP to discrete systems through appropriate discretization of the basis form. Higher-order parameterizations and discrete applications are tested and discussed thoroughly by Uygun (2000).

The Van de Vusse Reactor

In this section, the Van de Vusse reactor (1964) is used as the test case. The Van de Vusse reactor, shown in Fig.6, is a single-input single-output isothermal CSTR with two parallel/consecutive reactions of second-order kinetics. The conventional NMPC, local, global, and consecutive parameterization schemes are applied for the control of this system and compared with each other based on their control performances and computational loads.

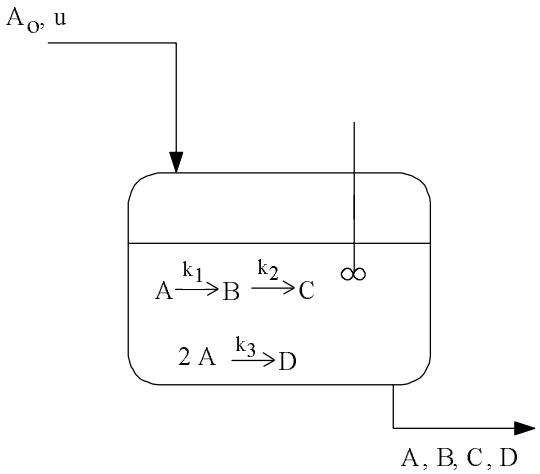


Figure 6. The Van de Vusse reactor and the reactions.

The modeling equations for the reactor system are given below:

$$\frac{dx_1}{dt} = -x_1 k_1 - k_3 x_1^2 + (x_{1f} - x_1)u \tag{19}$$

$$\frac{dx_2}{dt} = k_1 x_1 - k_2 x_2 + x_2 u \tag{20}$$

States x_1 and x_2 correspond to the concentrations (mol/L) of species A and B in the reactor. The feed concentration of species A is designated by x_{1f}. The manipulated variable, u, is the

dilution rate (L/hour). The rate constants are given as $k_1 = 50\ h^{-1}$, $k_2 = 100\ h^{-1}$, $k_3 = 10\ L\ mol^{-1}\ h^{-1}$. The nominal value of x_{1f} is 10 mol/L. The primary objective is to maintain the output variable state, x_2, at a setpoint of 1.0 mol/L. The objective function (Meadows and Rawlings, 1997) is given as:

$$\underset{u(k+1),\ldots,u(k+M)}{\text{minimize}} \quad \Phi\big(u(k),\ldots,u(k+M)\big) = \int_{t_k}^{t_{k+R}} \big(x_2(t)-1\big)^2\, dt \tag{21}$$

The dynamics and control of this reactor were studied by several researchers, including Kravaris (1990) and Sistu and Bequette (1995). The Van de Vusse reactor presents a challenge for process control because of several interesting properties of the system. The reactor exhibits input multiplicities, such that there are two steady-state solutions due to the quadratic term in the reaction kinetics. The two solutions for this system are {2.5, 1.0, 25} and {6.6667, 1.0, 233.33}, respectively for x_1, x_2, and u values (Meadows and Rawlings, 1997). Input multiplicities may cause a "sudden destabilization" of the system controlled with reset action (Sistu and Bequette, 1995). A system with input multiplicities is difficult to control because there exists more than one steady-state manipulated-variable value associated with a given output. In this situation, there is a possibility of the system to reach a steady-state different from the desired operation point. It is shown by Sistu and Bequette (1995) that in the presence of input multiplicities, the linearized model of a single-input single-output nonlinear system will always have Right-Half-Plane (RHP) zeros, provided that there is no simultaneous change in the relative order of the linearized process, and that the linearized-process dynamics are described by a strictly-proper transfer function with at least one finite zero. The Non-Minimum-Phase (NMP) behavior caused by RHP zeroes poses a difficult control problem because of unstable dynamics of the model inverse. In the case of linear NMP systems, the model is decomposed explicitly into invertible and all-pass portions and the invertible portion are used for control-law formulation. However, for nonlinear systems this type of decomposition technique is not available for the general case. In a linear predictive-control framework, a NMP system can be stabilized by using sufficiently long prediction horizons along with small control horizons (Maurath, 1988). For nonlinear predictive control no such results are available. Furthermore, the RHP zeros may indicate inverse-response behavior. This phenomenon can be observed in the response curves of the Van de Vusse Reactor, as depicted in Fig.7.

Inverse-response systems may cause sudden destabilization by classical feedback controllers due to the process-gain sign change. Sistu and Bequette (1995) demonstrate complications encountered in inverse-response systems, presenting the Van de Vusse reactor as one of the examples. Their simulations using Nonlinear Internal Model Control (NLIMC) exhibit unstable behavior.

A convenient method for dealing with a nonlinear system with multiple steady-states is the use of NMPC. Meadows and Rawlings (1997) discuss the use of NMPC for Van de Vusse reactor which produces promising control efficiency. The model incorporated within the NMPC structure enables the detection of the inverse-response exhibited by the observed variable, and computes the manipulated-variable profile accordingly. However, the prediction horizon should be sufficiently long such that the nonlinear behavior can be captured and consequences remedied by the optimizer.

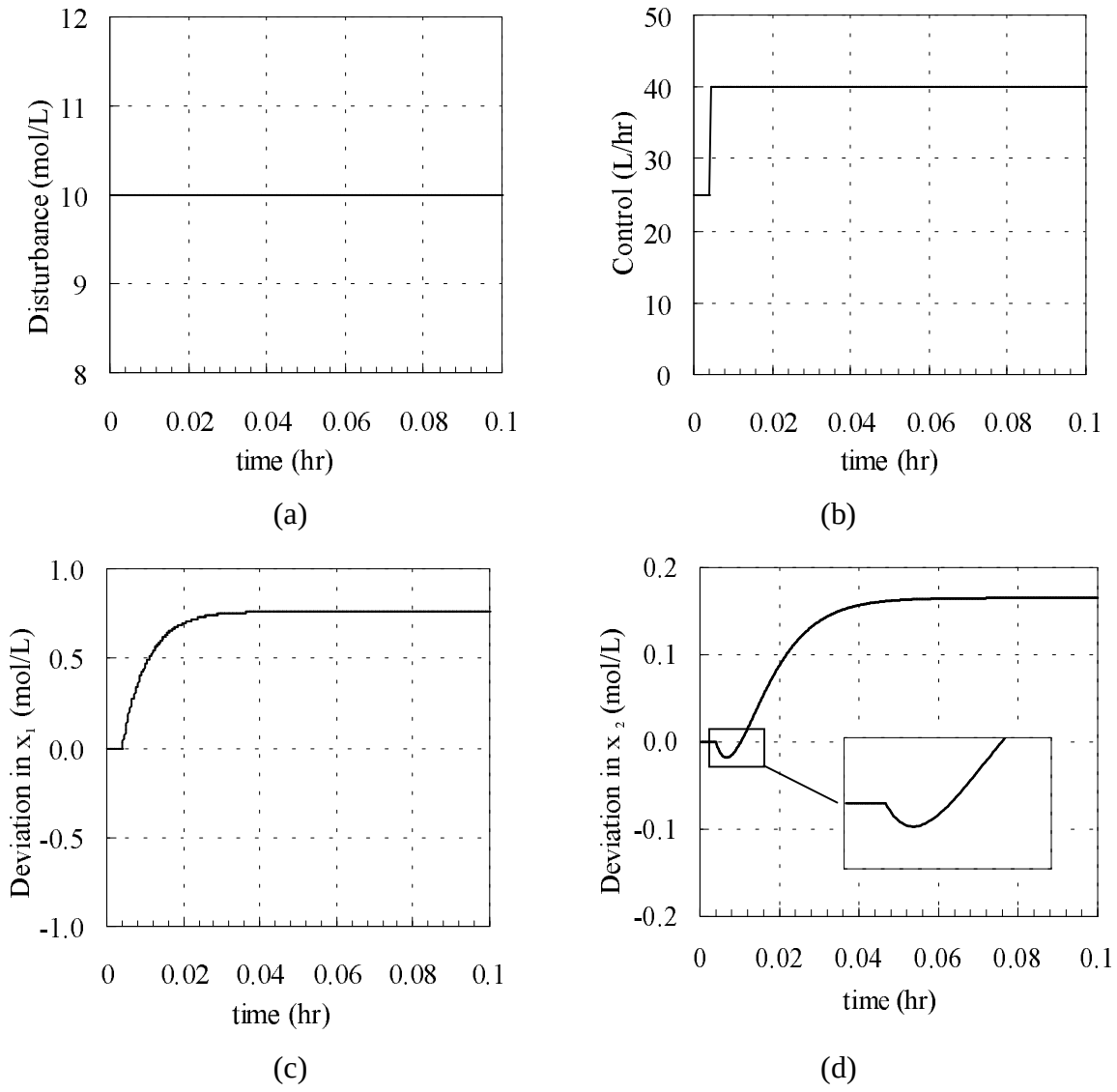


Figure 7. System response to increase in manipulated variable with no external disturbance: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

Numerical-Solution Procedure

A two-stage sequential approach was preferred to solve the constrained nonlinear dynamic-optimization problem defined in the NMPC-PCCVP scheme. In the sequential approach, the optimization routine serves as an outer loop to iteratively select new sets of manipulated-variable adjustments (or new sets of parameterization parameters). An ODE solver in the inner loop is used to integrate the dynamic equations at each iteration of the optimization.

Since a continuous-time-domain approach is utilized, the path-constraints for state and manipulated variables are converted into terminal-time constraints (Li and Biegler, 1988) as exemplified in Eqs.22 and 23 for the absolute manipulated-variable constraints.

$$\frac{dz}{dt} = \left\| \min\left\{0, \mathbf{u}_{k+j}(t) - \mathbf{u}^{min}\right\} \right\|^2 + \left\| \min\left\{0, \mathbf{u}^{max} - \mathbf{u}_{k+j}(t)\right\} \right\|^2$$

$$j=1,\ldots,R \quad t \in [t_k, t_{k+R}] \qquad (22)$$

$$z(0) = 0 \qquad (23)$$

Eq.22 integrates the violations of the constraints. Since the initial condition given by Eq.23 is zero, if no violations occurs, the artificial state z remains equal to zero. This artificial state is non-zero if and only if the constraints are violated. Therefore the terminal-time equality constraint, Eq.24, requires that the path-constraints are not violated.

$$z\left(t_{k+R}\right) = 0 \qquad (24)$$

The Control Problem

The control-performance comparison is done by observing the disturbance-rejection capabilities and relative-computation-time requirements of each NMPC-PCCVP scheme listed in Table 1. All schemes have a prediction horizon of seven, and control horizon of three steps. The disturbance is applied as a 10% increase in the input flow-rate. The NMPC simulations are run for 0.1 hours on all runs. Sampling frequency is taken to be 0.002 hours (7.2 seconds), though it is changed in investigation of the effects of sampling time.

Table 1. NMPC-PCCVP schemes investigated for the Van de Vusse reactor.

NMPC Scheme	Abbreviation
Conventional NMPC	NMPC
Local-Linear Parameterization	LLNMPC
Global-Linear Parameterization	GLNMPC
Consecutive-Linear Parameterization	CLNMPC

The manipulated-variable constraints are defined as: Δu^{max} = 5 L/hr and u^{min} = 15 L/hr, u^{max} = 30 L/hr. A state constraint on state one is imposed such that its deviation from the setpoint is limited to ±0.1 mol/L.

Results – The Perfect Model Case

The response curves under various predictive control schemes are depicted in Figs.8-11, and the computational requirements are presented in Fig.12.

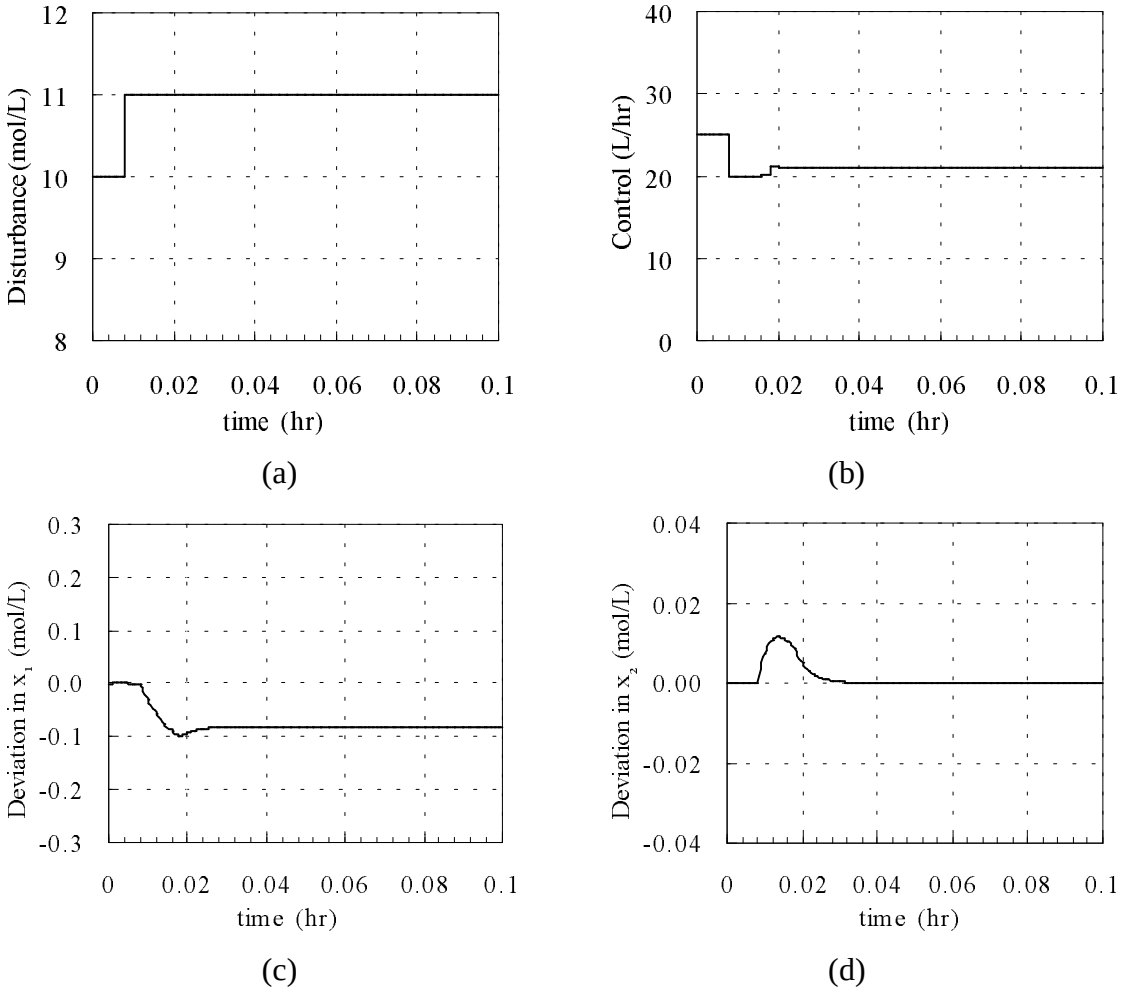


Figure 8. System responses under NMPC with perfect model: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

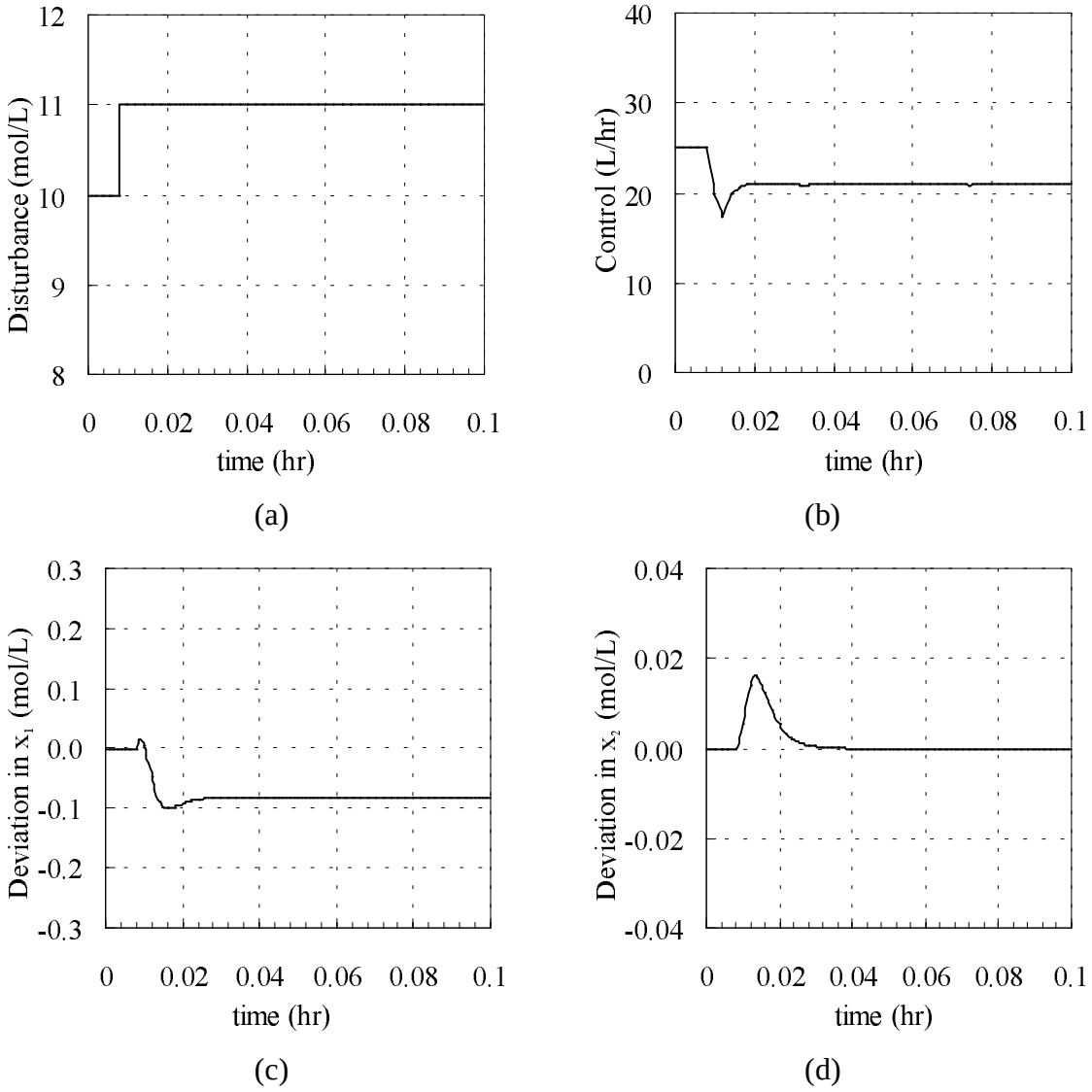


Figure 9. System responses under LLNMPC with perfect model: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

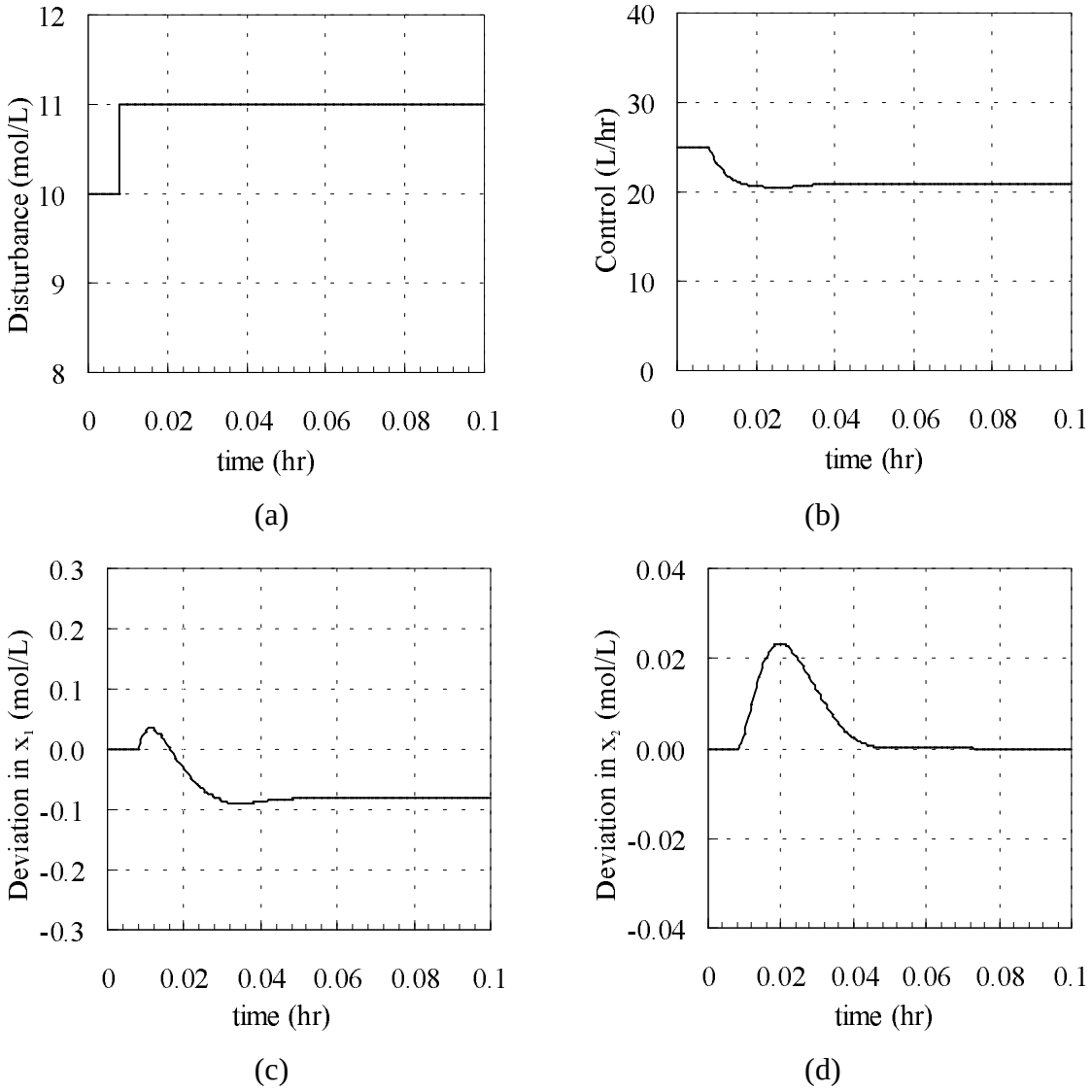


Figure 10. System responses under GLNMPC with perfect model: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

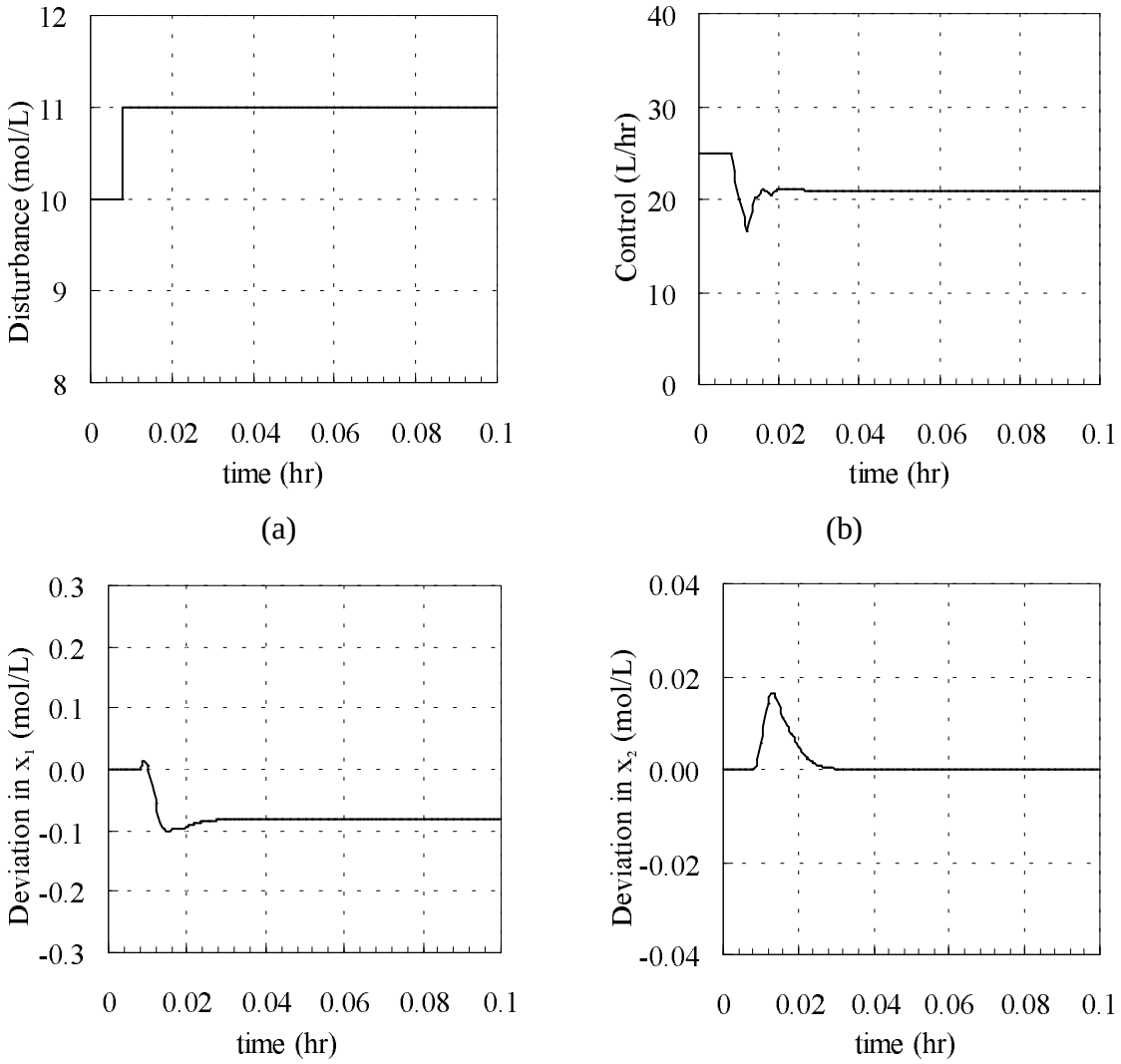


Figure 11. System responses under CLNMPC with perfect model: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

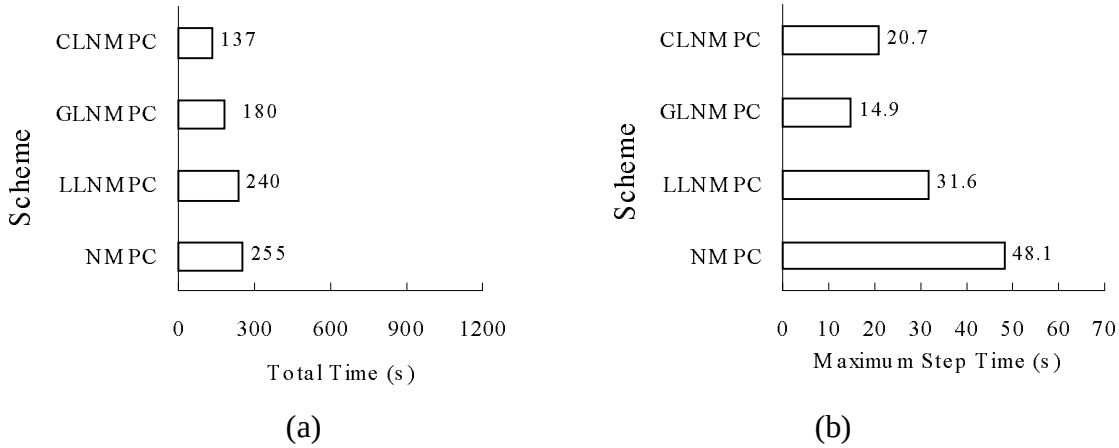


Figure 12. Computation-time comparison for NMPC with perfect model. a) total computation time, b) maximum time in a single step.

Results – The Plant/Model Mismatch Case

To test the limits of NMPC-PCCVP schemes in a worst-case scenario, a plant/model mismatch was introduced to the system where the state constraint was posed such that the plant cannot reach the setpoint without violating the constraint limits of ±0.1 mol/L imposed on state one. The mismatch was introduced so that the real process rate constants at their nominal values of $k_1 = 50\ h^{-1}$, $k_2 = 100\ h^{-1}$, $k_3 = 10\ L\ mol^{-1}\ h^{-1}$ were changed as $k_1 = 52\ h^{-1}$, $k_2 = 102\ h^{-1}$, $k_3 = 11\ L\ mol^{-1}\ h^{-1}$. Note that the mismatch was introduced at time zero and the disturbance was given at 0.008 hours. Therefore the controller must handle the mismatch and disturbance simultaneously. The mismatch scenario was run with the conventional NMPC (Fig.13) and GLNMPC (Fig.14). The computational requirements are presented in Fig.15.

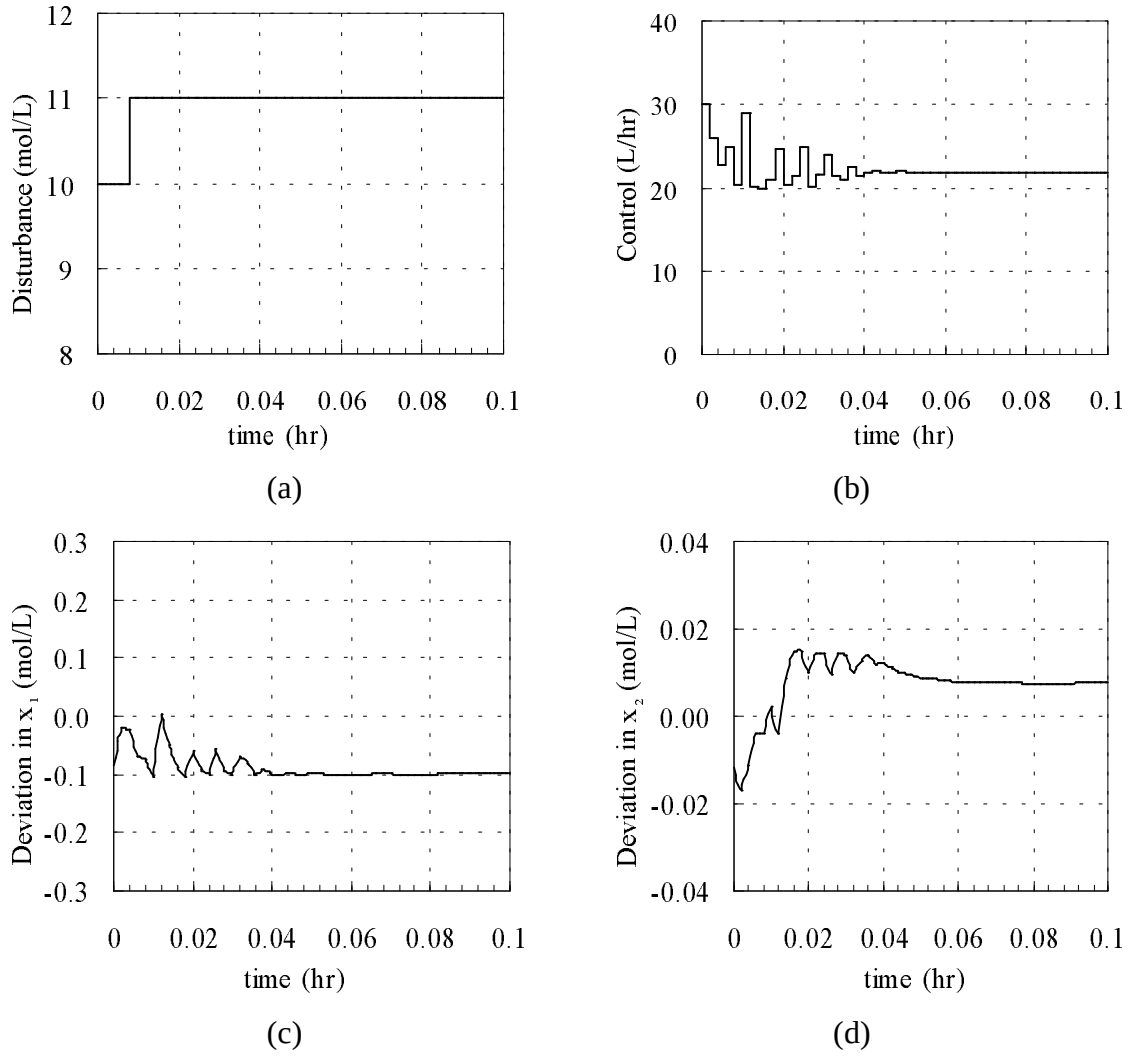


Figure 13. System responses under NMPC with plant/model mismatch: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

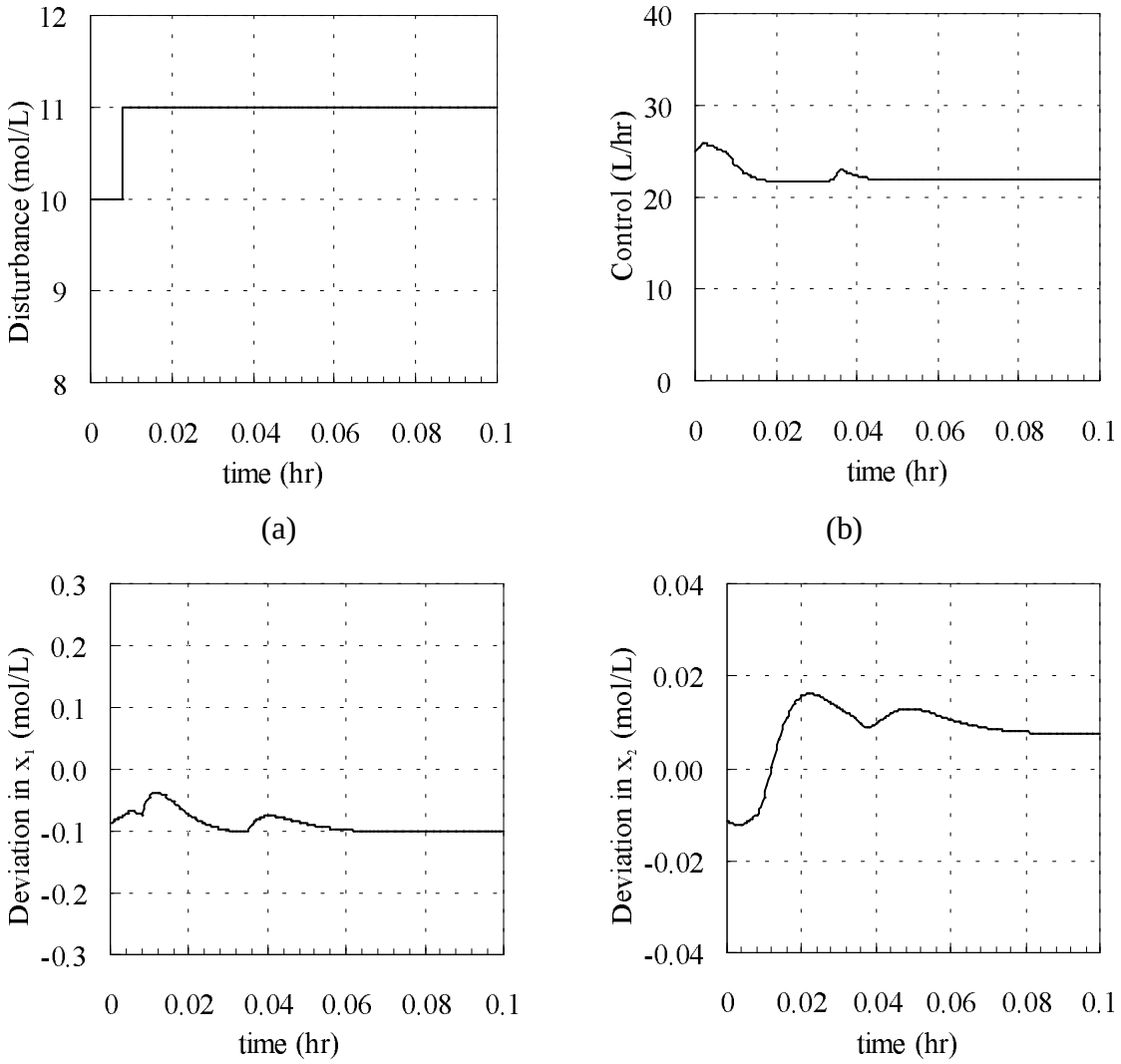


Figure 14. System responses under GLNMPC with plant/model mismatch: a) disturbance applied to the system, b) manipulated-variable profile, c) deviation from setpoint for state one, d) deviation from setpoint for state two.

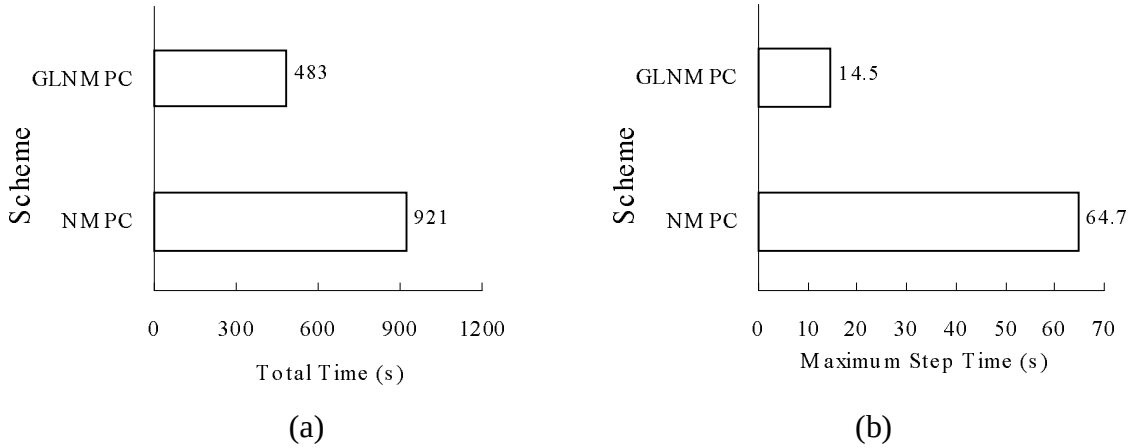


Figure 15. Computation-time comparison for NMPC with plant/model mismatch. a) total computation time, b) maximum time in single step.

Discussion

The real-time applicability of a NMPC scheme depends on both the maximum computation time in a step, and the total computation time. Ideally, optimization at each step should be completed within the computation interval. However, it is possible to bypass the current step, either by using the decision variables of the previous step (extending the parameterization of the previous step to the current step), or implementing the best decision-variable set evaluated up to the end of the computation interval. The deterioration of the control effectiveness would be tolerable provided that this bypass action is not applied excessively. A more severe limitation to be obeyed by each NMPC scheme is the total computation time which has to be completed within the simulation time (360 seconds for the Van de Vusse reactor). Violations of this limit imply that the per-step limits are very frequently violated, and prevent implementation on real-time. Thus, for detailed comparison of the computational performances, both the total computation time and the per-step computation time results were presented in this work.

As it may be observed in the resulting profiles, the conventional NMPC scheme produces very efficient control with the perfect model, though calculation time requirements are prohibitive with respect to the PCCVP schemes. The LLNMPC exhibits a reduction in the computation time with control performance matching that of the conventional NMPC although the number of decision variables is the same. The GLNMPC scheme produces the most rapid computations, compensating the relatively higher overshoot observed. As expected, the CLNMPC scheme reduces the magnitude of the overshoot at the cost of somewhat higher maximum per step computation intervals. The significant difference between the computation-time requirements of the conventional NMPC and the PCCVP schemes favors the control-vector-parameterization applications.

For the perfect model case, the per-step computation times reflect that the GLNMPC scheme decreases the computation-time requirement up to a third of the requirements under NMPC. The CLNMPC scheme provides faster computation with respect to the NMPC and LLNMPC. The total computation times conform to the same order of computational efficiency. The LLNMPC and NMPC are observed to produce more aggressive controls which yield a faster settling of the output to the setpoint in perfect-model situations, though the aggressive action of NMPC produces a fluctuating response under plant/model mismatch. The GLNMPC scheme produces very smooth and efficient control on all cases, with the fastest computation time, and is observed to be robust to plant/model mismatch.

The plant/model mismatch is introduced as a worst-case scenario. The state constraint is posed such that the plant cannot reach the setpoint without violating the constraint. As it can be observed in Fig.13, this case causes an erratic controller behavior with the conventional NMPC, causing a kinky response in the observed variable. Contrary to the conventional NMPC scheme, the GLNMPC settles smoothly to the constraint limit, and is twice as fast as NMPC computation-wise.

Conclusions

In this work, the potential use of the PCCVP technique for a computationally efficient NMPC algorithm was investigated. In addition to the LLNMPC and GLNMPC schemes, the CLNMPC scheme was designed to overcome difficulties with inverse-response systems.

With respect to the computation requirements, the GLNMPC is the fastest scheme, followed by the CLNMPC. It was observed that the LLNMPC was faster with respect to the conventional NMPC although the number of decision variables are the same.

The results indicate that the most successful scheme is the GLNMPC and it should be preferred over the classical NMPC scheme. This scheme produces smoother control with reduced computation-time requirements, and is more robust against plant/model mismatches, and hence should be preferred.

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Nomenclature

$\mathbf{c}_h(k+m)$	h^{th} parameterization parameter vector at step k+m
$\mathbf{d}$	disturbance vector
$\mathbf{d}_k^{p/m}$	plant/model mismatch vector, measured at step k
h	identifier for parameterization parameter
H	order of parameterization
k	current time step
m	control horizon step identifier
M	control horizon length
$\mathbf{Q}$	weighing matrix
R	prediction horizon
t	time
t_k	time at start of step k
$\mathbf{u}$	vector of manipulated variables
$\mathbf{u}_{k+m}$	vector of manipulated variables at step k+m
$\mathbf{u}^{max}$	maximum values for the vector of manipulated variables
$\mathbf{u}^{min}$	minimum values for the vector of manipulated variables
$\Delta\mathbf{u}$	change in manipulated variables
$\Delta\mathbf{u}^{max}$	maximum values for the change in vector of manipulated variables
$\Delta\mathbf{u}^{min}$	minimum values for the change in vector of manipulated variables
$\mathbf{x}$	vector of state variable
$\mathbf{x}^{max}$	maximum values for the vector of state variables
$\mathbf{x}^{min}$	minimum values for the vector of state variables
$\mathbf{x}^{MP}$	vector of state variables, predicted by the model
$\mathbf{x}^{P}$	vector of state variables, observed from the plant

$\mathbf{x}^{set}$	vector of set points for state variables
z	artificial state
Φ	objective function in dynamic optimization
τ	shifted time

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