Simultaneous MILP Synthesis with Flexibility Targeting of Minimum-Utility Heat-Exchanger Networks and Minimum-Cost Flexible Retrofit Design

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Abstract – A two-stage methodology for the design of heat exchanger networks with desired steady-state operability/controllability range at the least cost is proposed. In the first stage, the synthesis of energy-optimal structures satisfying a given flexibility-index target is accomplished through a vertex-based one-stage, non-iterative, super-structure-based simultaneous MILP formulation. In the second stage, the energy-optimal flexible structures generated in the first stage are designed via a vertex-based one-stage NLP formulation to obtain total-cost-optimal flexible retrofit designs.

Introduction

One of the most studied problem in chemical engineering is the synthesis/design of heat exchanger networks (HENs) which was originally defined as: "*given nh hot streams to be cooled and nc cold streams to be heated from specified supply temperatures to specified target temperatures, design the network of heat exchangers, heaters and coolers accomplishing this task at the least cost*" (Masso and Rudd, 1969). Over the past three decades, many heuristic, evolutionary, and algorithmic approaches for the solution to the problem were developed. Due to the triple trade-off among utility consumption, total heat-transfer area, and the number of heat-exchange units, many solution techniques proposed to decompose the problem into stages (Nishida et al. 1977; Linnhoff and Flower 1978a, 1978b). These stages are mainly: i) *targeting* of minimum utility consumption (and/or minimum heat-transfer area) and minimum number of units (exchangers, heaters and coolers), ii) *synthesis* of a HEN structure with minimum utility consumption and with minimum or close to minimum number of units, iii) *evolution* of the synthesized structure using heuristics or optimization techniques to get a less expensive network design or to get a network design that provides specific requirements such as operability, flexibility, and safety at the same time. Superstructure based mathematical programming approaches aiming to find global optimum in a single step were also introduced in the late 80`s (Floudas et al. 1986; Yee and Grossmann 1990a,b).

The solution techniques offered up to now for the heat-integration problem of chemical processes have been concentrated on mostly around the standard definition of the problem as stated above for the fixed operating conditions. However, in a real process, many flows and temperatures are variable and even it may be requested to change the process conditions. These situations may cause not only a decrease in the energy-recovery efficiency and economical performance of the process but also risk the other process equipments integrated

to the heat-recovery system unless the HEN is operable under the expected/unexpected disturbances. The task of finding the operability limits of a process was introduced with the Flexibility Index definition by Swaney and Grossmann (1985). The flexibility index gives a measure of the size of the region of feasible operation in the space of deviations of the uncertain parameters (generally source temperatures for HENs) from their nominal values. For a given HEN problem, the flexibility index of the HEN structure (obtained using any of the solution techniques briefly mentioned above) can be calculated by applying thermodynamic constraints. However, for the calculation of the flexibility index of a HEN after determining its structure, there are two questions: i) If the resulting network structure does not allow the flexibility desired by the designer, how can the structure be modified so that the desired flexibility (operability range) is obtained?, ii) If these modifications allow the synthesis of a network structure with the desired flexibility measures, is this network the least cost solution for the specified flexibility measures?

In this work, a two-stage procedure is proposed to design a HEN that has desired operability range (defined by the vertex-based flexibility index) at the least cost for a given HEN problem. In the first stage of the methodology proposed, the energy-optimal structures with specified flexibility (flexibility index) are generated via a non-iterative, one-stage, superstructure-based simultaneous MILP formulation. In the second stage, the energy-optimal flexible structures obtained in the first stage are designed via one-stage NLP formulation to obtain cost-optimal flexible retrofit designs.

General Formulation for the Generation of Energy-Optimal Flexible HEN Structures (Stage 1)

The general Mixed-Integer-Programming (MIP) formulation for a superstructure-based simultaneous HEN synthesis/design and flexibility targeting can be stated as (Konukman et al., 1999):

$$\min_{\mathbf{d},\mathbf{x},\mathbf{u},\mathbf{z}} \ \Phi(\mathbf{d},\mathbf{x},\mathbf{u},\mathbf{z},\mathbf{p},\boldsymbol{\theta}) \tag{P1.1}$$

$$\text{s.t.} \ \ \mathbf{h}(\mathbf{d},\mathbf{x},\mathbf{u},\mathbf{z},\mathbf{p},\boldsymbol{\theta}) = \mathbf{0} \tag{P1.2}$$

$$\mathbf{g}(\mathbf{d},\mathbf{x},\mathbf{u},\mathbf{z},\mathbf{p},\boldsymbol{\theta}) \le \mathbf{0} \tag{P1.3}$$

$$\theta \in R(\delta) = \left\{\theta \,\middle|\, \left(\theta^o - \delta\Delta\theta^-\right) \le \theta \le \left(\theta^o + \delta\Delta\theta^+\right)\right\} \tag{P1.4}$$

where **d** is the vector of design variables (exchanger areas), **x** is the vector of state variables (outlet temperatures), **z** is the vector of binary variables (denoting potential existence/absence of exchangers embedded in the superstructure), **u** is the vector of control/free variables (heater/cooler loads), **p** is the vector of fixed parameters (heat-transfer coefficients), and $\boldsymbol{\theta}$ is the vector of uncertain parameters (in this work, source-stream temperatures). Eq. P1.1 is the objective function and can be formulated in terms of total annualized cost or total utility consumption. For a synthesis problem, the vector **d** is dropped from the set of decision variables. Eqs.P1.2- P1.3 are the vector of equality and inequality constraints. If the vector of states, **x**, can be eliminated using Eq. P1.2, then Eq. P1.3 can be written as the vector of reduced inequalities. Eq. P1.4 describes the range of the uncertain parameters as deviations from nominal values, θ^o, where $\Delta\theta^-$ and $\Delta\theta^+$ are the scaled deviations from θ^o, and the non-negative scalar, δ, is the targeted flexibility. The region $R(\delta)$ describes the scaled hyperrectangle which is required to be within the feasible region defined by Eqs. P1.2- P1.3.

For a fixed HEN topology and design (known **z** and **d**), the flexibility index, δ^*, defined by Swaney and Grossmann (1985) is the maximum of scalar δ. This problem (Eqs. P1.1- P1.4) can be solved indirectly with a two-stage procedure (as iterations between a multiperiod synthesis problem and a flexibility-analysis step (Papalexandri and Pistikopoulos, 1993), however, this technique requires the solution of large MIPs through, for example, the Generalized Benders Decomposition (Floudas, 1995) in each iteration.

At this point, the synthesis of minimum-utility HEN structures satisfying a given flexibility target will be accomplished through a one-stage, non-iterative, superstructure-based formulation with the supposition that the feasible region defined by the reduced inequality constraints is convex in uncertain parameters. Therefore, the optimal solution can explored on the basis of the vertices of the polyhedral region of uncertainty. For this purpose, the general vertex-based formulation is described by (Konukman et al., 1999):

$$\min_{\mathbf{x}^n,\mathbf{z}^o,\mathbf{u}^n} \quad \Phi\left(\mathbf{d}^o,\mathbf{x}^o,\mathbf{u}^o,\mathbf{z}^o,\mathbf{p},\boldsymbol{\theta}^o\right) \qquad n = 0, 1, ..., 2^N \qquad \text{(P1.5)}$$

$$\text{s.t.} \quad \mathbf{h}^n\left(\mathbf{d}^o,\mathbf{x}^n,\mathbf{z}^o,\mathbf{u}^n,\mathbf{p},\boldsymbol{\theta}\right)=\mathbf{0} \qquad n = 0, 1, ..., 2^N \qquad \text{(P1.6)}$$

$$\mathbf{g}^n\left(\mathbf{d}^o,\mathbf{x}^n,\mathbf{z}^o,\mathbf{u}^n,\mathbf{p},\boldsymbol{\theta}\right)\le\mathbf{0} \qquad n = 0, 1, ..., 2^N \qquad \text{(P1.7)}$$

$$\boldsymbol{\theta}-\boldsymbol{\theta}^o=\mathbf{0} \qquad n = 0 \qquad \text{(P1.8)}$$

$$\boldsymbol{\theta}-\boldsymbol{\theta}^o-\delta\boldsymbol{\theta}^n=\mathbf{0} \qquad n = 1, ..., 2^N \qquad \text{(P1.9)}$$

$$\theta^n \in R_o(\delta)=\left\{\theta^n \,\middle|\, \theta^n \text{ is a vertex of } R(\delta), n = 1,...,2^N\right\} \qquad \text{(P1.10)}$$

The vertex formulation, described by Eqs. P1.5- P1.10, is a reformulation of the more general non-vertex formulation, described by Eqs. P1.1- P1.4, using the approach named as "Algorithm I" by Halemane and Grossmann (1983). In the case the feasible region defined by the reduced inequality constraints is convex in uncertain parameters, θ, the formulations described by Eqs. P1.1- P1.4 and Eqs. P1.5- P1.10 are identical (Swaney and Grossmann,1985; Halemane and Grossmann, 1983). For a HEN problem where the uncertain parameters are considered to be the source-stream temperatures only, the feasible region defined by the reduced inequality constraints is convex (Floudas, 1995). Thus, the critical point that limits the solution lies at a vertex of the polyhedral region of uncertainty, and Eqs. P1.5- P1.10 can be used instead of Eqs. P1.1- P1.4. Eq. P1.5 is the objective function that is evaluated at the nominal operating point, *n=0*. Eqs. P1.6- P1.7 are the augmented forms of the vector of equality and inequality constraints (Eqs.2-3) that are evaluated at the nominal operating point (*n=0*) as well as at each vertex (*n=1,...,*2^N) simultaneously. The θ values in Eqs. P1.6- P1.7 are governed by Eqs. P1.8- P1.9. Eq. P1.9 constrains the vector θ to take only 2^N vertex values, θ^n (2^N $\pm$ directed deviations from the nominal, θ^o, values), scaled by the scalar, δ, which is the desired flexibility. Eq. P1.8 denotes that at *n=0*, δ is considered to be zero. The set $R_o(\delta)$ describes the vertices of the scaled hyperrectangle which is required to be within the feasible region defined by Eqs. P1.6- P1.7. As indicated in Eq. P1.5, a unique set of the binary decision variables, $\mathbf{z}^o$, controlling the structural modifications, applies to all 2^N+1 operating points (2^N vertex points plus the nominal operating point). This enforces the optimal structure to be reduced from the superstructure to be valid at all 2^N+1 operating points. The state variables, $\mathbf{x}^n$ (e.g., the exchanger outlet temperatures), and the control/free variables, $\mathbf{u}^n$ (e.g., heater/cooler and exchanger heat loads), are the vectors of continuous decision variables defined for all 2^N+1 operating points. In this way, the optimal structure which is unique to all operating points and the optimal values of the operating-point-dependent state and control

vectors are obtained simultaneously as the solution of a single mixed-integer optimization problem defined by Eqs. P1.5- P1.10.

The specific form, and the total number of decision variables and constraints of the general vertex-based "simultaneous synthesis and flexibility-index targeting" formulation described by Eqs. P1.5- P1.10 depends on the particular superstructure representation of the process. At the first stage of the methodology proposed here, Eqs. P1.5- P1.10 are applied to the HEN superstructure proposed by Yee and Grossmann (1990a,b). The specific formulation presented here is based on the simultaneous-MINLP model developed by Yee and Grossmann (1990a,b) as a solution to the standard HEN-synthesis problem. The original MINLP formulation (1990a,b) (a nonlinear total-cost function as objective and a set of linear constraints) is based on a stagewise superstructure representation that considers potential exchanges between any pair of hot and cold streams within a stage via binary decision variables. The model of Yee and Grossmann (1990a,b) is reformulated as MILP, by considering the minimum total-utility consumption as the linear objective function, and the set of linear constraints is augmented for each vertex such that the resulting HEN structure satisfies expected deviations, with magnitude δ, in source-stream temperatures in every combination of disturbance directions (vertex directions) which are identified by an additional index *n*. In other words, the superstructure is forced to yield a unique minimum-utility structure that is operable at the nominal condition and at each vertex of the feasible region the size of which is determined by the desired flexibility index. The nominal operating condition at the centeroid of the polyhedral region of uncertainty is identified by $n=0$. For a HEN problem with N_H hot streams and N_C cold streams, there are $N=2^{(N_H+N_C)}$ vertices (+ and − directed simultaneous disturbance directions) besides the nominal operating point (under no disturbance). Overall heat balances for each stream, heat balances at each stage of the superstructure, assignment of superstructure inlet temperatures, equations for the feasibility of temperatures, hot-/cold-utility-load equations, logical constraints, and approach-temperature equations forming the set of linear constraints in the original formulation of Yee and Grossmann (1990a,b) are all written for each of the $2^{(N_H+N_C)}$ vertices as well as for the nominal operating point. Source-stream temperatures are perturbed for each vertex by a fixed scalar, δ, which is the targeted flexibility. Thus, satisfaction of the linear constraints is required for each vertex and for the nominal operating point. The target temperatures specified by the HEN problem are required to be matched exactly for each vertex by variable (vertex-dependent) heater/cooler and exchanger heat loads. The objective function is the minimization of the total utility consumption at the nominal operating point; $n=0$. Thus, the whole problem becomes a MILP for which the superstructure-dependent global solution can be obtained.

Formulation of the Optimal Flexible Retrofit Design for Specified HEN Structures (Stage 2)

Using the formulation described in Stage 1, flexible and energy-optimal structures are obtained. The second stage of the strategy proposed in this work is the design/retrofit (size the areas of process/utility exchangers, and decide on the location of bypasses) of the energy-optimal flexible structures obtained in the first stage so that they satisfy the desired flexibility at the least cost. At this point, the difference between the *structural flexibility* and *design flexibility* of a HEN should be mentioned. A HEN structure gains its flexibility via operating limits of heaters and coolers and location of bypasses which directly affect the size (area) of exchangers. Thus, without considering the areas of exchangers, the flexibility-index calculation of a HEN will indicate only the thermodynamic limits of that structure. On the

other hand, in the calculation of the flexibility index of a HEN with sized exchangers, the only flexibility sources are variable/controllable heater and cooler loads. Thus, the flexibility index of a designed HEN (design flexibility index) is always smaller than or (at most) equal to the flexibility index of its structure. In another point of view, for a specified HEN structure, the desired flexibility may be gained by adjusting the exchanger areas in an optimization framework at the design stage. The desired flexibility of a HEN will be defined in the following formulation as the *retrofit-design flexibility index* (δ_{RD}). The task of designing a HEN (retrofit design), which is not only economically at minimum cost but also has the ability to reject the disturbances predefined in *all* disturbance directions (i.e., δ_{RD}), can be introduced as a single NLP problem. The equality constraints (energy balances for exchangers and mixing points) and inequality constraints (heat loads for exchangers and utilities, minimum-approach temperatures for exchangers) for each disturbance direction can be evaluated in the same problem. The only common variables of each disturbance set are the heat-exchanger areas which must be valid for all disturbance directions. In the formulation, the nominal operating point is also considered as a direction. The effect of the nominal-direction on the overall cost of the resulting optimal retrofit design is profound since the utility consumptions are taken into account only in the nominal-direction. The general formulation of the vertex-based retrofit-design optimization problem for a specified retrofit design flexibility index (δ_{RD}) can be described with the following form (Konukman, 1996):

$$\underset{\begin{pmatrix}\mathbf{u}_h^i,\mathbf{u}_c^i,\mathbf{x}^i,\mathbf{T}_{he}^i,\mathbf{T}_{ce}^i\\ \mathbf{A}_j,\mathbf{A}_k,\mathbf{A}_l,\mathbf{Q}_h^0,\mathbf{Q}_c^0\end{pmatrix}}{\text{minimize}} \quad \Phi(\mathbf{u}_h^i,\mathbf{u}_c^i,\mathbf{x},\mathbf{T}_{he}^i,\mathbf{T}_{ce}^i,\mathbf{A}_j,\mathbf{A}_k,\mathbf{A}_l,\mathbf{Q}_h^0,\mathbf{Q}_c^0) \qquad \text{(P2.1)}$$

subject to:

equality constraints:

$$\mathbf{h}(\mathbf{u}_h^i,\mathbf{u}_c^i,\mathbf{x}^i,\mathbf{T}_{he}^i,\mathbf{T}_{ce}^i,\mathbf{A}_j,\mathbf{A}_k,\mathbf{A}_l,\mathbf{Q}_h^0,\mathbf{Q}_c^0)=\mathbf{0} \qquad (2^{NS}+1)\times(2\times NE+NHT)+2\times(NC+NH) \qquad \text{(P2.2)}$$

inequality constraints:

$$\mathbf{g}(\mathbf{u}_h^i,\mathbf{u}_c^i,\mathbf{x}^i,\mathbf{T}_{he}^i,\mathbf{T}_{ce}^i,\mathbf{A}_j,\mathbf{A}_k,\mathbf{A}_l,\mathbf{Q}_h^0,\mathbf{Q}_c^0)\le\mathbf{0} \qquad (2^{NS}+1)\times(4\times NE+NC+NH+NST) \qquad \text{(P2.3)}$$

side constraints:

$$u_h^{min}\le\mathbf{u}_h^i<u_h^{max} \qquad NHB\times(2^{NS}+1) \qquad \text{(P2.4)}$$

$$u_c^{min}\le\mathbf{u}_c^i<u_c^{max} \qquad NCB\times(2^{NS}+1) \qquad \text{(P2.5)}$$

$$x^{min}\le\mathbf{x}^i<x^{max} \qquad NX\times(2^{NS}+1) \qquad \text{(P2.6)}$$

In this formulation, the decision variables are $\mathbf{u}_h^i$ (vector of hot-side bypass fractions for all disturbance directions), $\mathbf{u}_c^i$ (vector of cold-side bypass fractions for all disturbance directions), $\mathbf{x}^i$ (vector of split fractions, if present, for all disturbance directions), $\mathbf{T}_{he}^i$ (vector of hot-side outlet temperatures for all disturbance directions), $\mathbf{T}_{ce}^i$ (vector of cold-side outlet temperatures for all disturbance directions), $\mathbf{A}_j$ (vector of heat-exchanger areas common for all disturbance directions), $\mathbf{A}_l$ (vector of heater areas for zero-disturbance direction), $\mathbf{A}_k$ (vector of cooler areas for zero-disturbance direction), $\mathbf{Q}_h^0$ (vector of heater duties, if present,

for zero disturbance direction), $\mathbf{Q}_c^0$ (vector of cooler duties, if present, for zero disturbance direction). The total number of decision variables is $(2^{NS}+1)\times(NCB+NHB+NX+2\times NE)+NE + 2\times(NC+NH)$. The vector of equality constraints, **h**, is formed by the energy-balance equations (two per exchanger for all disturbance directions), hard-target constraints for all disturbance directions (if present), energy balance equation for each heater (two per heater), and energy balance equation for each cooler (two per cooler) for zero-disturbance direction. The total number of such equality constraints is $(2^{NS}+1)\times(2\times NE+NHT)+2\times(NC+NH)$. The vector of inequality constraints, **g**, is formed by the heat-load constraints for exchangers (two per exchanger), for all disturbance directions, approach-temperature constraints for exchangers (two per exchanger) for all disturbance directions, heat-load constraints for heaters and/or coolers (if present) for all disturbance directions, and by the soft-target (range) constraints (if present) for all disturbance directions. The total number of such inequality constraints is $(2^{NS}+1)\times(4\times NE+NC+NH+NST)$. The vector of side constraints is formed by the upper and lower bounds on the hot- and cold-side bypass fractions and on the split fractions for all disturbance directions. The total number of such side constraints is $(2^{NS}+1)\times(NCB+NHB+NX)$. In the explicit formulation given below, exchangers are taken as counter-current single-pass exchangers. Heat capacities of fluids and heat-transfer coefficients are assumed to be constant.

$$\underset{\left(\begin{array}{c}\mathbf{u}_{h,j}^{i,p},\,\mathbf{u}_{h,j}^{i,p},\,\mathbf{x}^{i,p}\\ \mathbf{T}_{he,j}^{i,p},\mathbf{T}_{ce,j}^{i,p},\mathbf{Q}_h^{0,p},\mathbf{Q}_c^{0,p}\\ \mathbf{A}_j,\,\mathbf{A}_l,\,\mathbf{A}_k\end{array}\right)}{\text{minimize}} \sum_{j=1}^{NE}\left(C_E+\alpha A_j^{\beta}\right)+\sum_{k=1}^{NC}\left(C_C+\alpha A_k^{\beta}\right)+\sum_{j=1}^{NH}\left(C_H+\alpha A_l^{\beta}\right) + \sum_{k=1}^{NC}\left(C_{CU}+Q_{c,k}^0\right)+\sum_{l=1}^{NH}\left(C_{HU}+Q_{h,l}^0\right) \qquad \text{(P2.7)}$$

For every disturbance direction (i) in all the source streams and for every exchanger (j) in the network, equality and inequality constraints are:

Energy-balance equations (equality constraints):

$$w_{h,j}^i\left(1-u_{h,j}^i\right)\left(T_{hi,j}^i-T_{he,j}^i\right)+w_{c,j}^i\left(1-u_{c,j}^i\right)\left(T_{ci,j}^i-T_{ce,j}^i\right)=0 \qquad i = 0,...,2^{NS},\quad j = 1,...,NE \qquad \text{(P2.8)}$$

$$w_{h,j}^i\left(1-u_{h,j}^i\right)\left(T_{hi,j}^i-T_{he,j}^i\right)+U_jA_j\Delta T_{lm,j}^i=0 \qquad i = 0,...,2^{NS},\quad j = 1,...,NE \qquad \text{(P2.9.a)}$$

or

$$w_{c,j}^i\left(1-u_{c,j}^i\right)\left(T_{ci,j}^i-T_{ce,j}^i\right)-U_jA_j\Delta T_{lm,j}^i=0 \qquad i = 0,...,2^{NS},\quad j = 1,...,NE \qquad \text{(P2.9.b)}$$

where: $$\Delta T_{lm,j}^i=\frac{\left(T_{hi,j}^i-T_{ce,j}^i\right)-\left(T_{he,j}^i-T_{ce,j}^i\right)}{\ln\left[\left(T_{hi,j}^i-T_{ce,j}^i\right)\Big/\left(T_{he,j}^i-T_{ci,j}^i\right)\right]} \qquad i = 0,...,2^{NS},\quad j = 1,...,NE \qquad \text{(P2.10)}$$

The overall heat-transfer coefficients can be introduced as the property of streams:

$$U_j = \frac{1}{\left(1/h_{h,j}\right) + \left(1/h_{c,j}\right)} \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.11)}$$

For every mixing point after an exchanger, energy-balance equations if bypass exist:

$$u^i_{h,j} T^i_{hi,j} + \left(1 - u^i_{h,j}\right) T^i_{he,j} - T^i_{hf,j} = 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.12)}$$

$$u^i_{c,j} T^i_{ci,j} + \left(1 - u^i_{c,j}\right) T^i_{ce,j} - T^i_{cf,j} = 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.13)}$$

Load constraints (inequality constraints):

$$T^i_{he,j} - T^i_{hi,j} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.14)}$$

$$T^i_{ci,j} - T^i_{ce,j} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.15)}$$

Minimum approach-temperature constraints (inequality constraints):

$$T^i_{ce,j} - T^i_{hi,j} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.16)}$$

$$T^i_{ci,j} - T^i_{he,j} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.17)}$$

Bypass-ratio constraints (if bypasses exist) (side constraints):

$$0 \le u^i_{h,j} < 1 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.18)}$$

$$0 \le u^i_{c,j} < 1 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.19)}$$

Split-ratio constraints (if splits exist) (side constraints):

$$0 \le x^i < 1 \qquad i = 0,...,2^{NS} \qquad \text{(P2.20)}$$

Heater load constraints (inequality constraints):

$$T^i_{cf,j} - T^t_{c,l} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.21)}$$

Cooler load constraints (inequality constraints):

$$T^t_{h,k} - T^i_{hf,j} \le 0 \qquad i = 0,...,2^{NS},\ \ j = 1,...,NE \qquad \text{(P2.22)}$$

The heat loads of heaters for nominal direction (equality constraints):

$$Q^0_{h,l} - w^0_{c,j}\left(T^t_{c,l} - T^0_{cf,j}\right) = 0 \qquad l=1, \ldots, NH \qquad \text{(P2.23)}$$

$$Q^0_{h,l} - U_l A_l \Delta T^0_{lm,l} = 0 \qquad l=1, \ldots, NH \qquad \text{(P2.24)}$$

$$\text{where: } \Delta T^0_{lm,l} = \frac{\left(T^0_{cf,j} - T^0_{hte,l}\right) - \left(T^t_{c,j} - T^0_{hti,l}\right)}{\ln\left[\left(T^0_{cf,j} - T^0_{hte,l}\right) \Big/ \left(T^t_{c,j} - T^0_{hti,l}\right)\right]} \qquad l=1, \ldots, NH \qquad \text{(P2.25)}$$

The heat loads of coolers for nominal direction (equality constraints):

$$Q^0_{c,k} = w^0_{h,j}\left(T^0_{hf,j} - T^t_{h,k}\right) \qquad k=1, \ldots, NC \qquad \text{(P2.26)}$$

$$Q^0_{c,k} - U_k A_k \Delta T^0_{lm,k} = 0 \qquad k=1, \ldots, NC \qquad \text{(P2.27)}$$

$$\text{where: } \Delta T^0_{lm,k} = \frac{\left(T^0_{hf,j} - T^0_{cle,k}\right) - \left(T^t_{h,k} - T^0_{cli,k}\right)}{\ln\left[\left(T^0_{hf,j} - T^0_{cle,k}\right) \Big/ \left(T^t_{h,k} - T^0_{cli,k}\right)\right]} \qquad k=1, \ldots, NC \qquad \text{(P2.28)}$$

In order to complete the general formulation, the connections between the exchangers should be defined for every disturbance direction (i), the target temperatures of the network should be defined as hard (equality) or soft (inequality/range) constraints, and the temperatures of all the source streams entering the exchangers should be defined in terms of desired δ_{RD} values.

Example Problem

For the demonstration of the two-stage strategy proposed in this work, the HEN problem defined by Yee and Grossmann (1991) is studied. Stream conditions and economic parameters are as shown in Table 1. Using Eqs P1.5-P1.10, for different values of flexibility index, the energy-optimal flexible structures and utility-consumption values corresponding to these structures can be obtained. For this example, the three-stage HEN superstructure (Yee and Grossmann, 1991) is used. In Figure 1, the result of Stage 1 is summarized in terms of the flexibility index, utility-consumption level and the HEN structures. As can be seen from Figure 1, there are five plateaus of total-utility-consumption levels at the nominal operating point. In the first plateau, there is one structure (Structure A) with 2250 kW nominal total-utility requirement. This structure is also the pinch solution (with ΔT_{min} = 10 K) and can also be obtained by the algorithm of Yee and Grossmann (1991). The flexibility index of this structure is 37 K (i.e., Structure A is operable as long as the source-temperature variations are between 0 K - 37 K). In the second plateau, there is only one structure (Structure B) with a nominal total-utility-consumption level of 3183 kW, and the flexibility index of this structure is 74.2 K (i.e., Structure B is operable as long as the source-temperature variations are between 0 K - 74.2 K). There are three other plateaus (with 3750 kW, 4250 kW, and 8850 kW nominal total-utility-consumption levels), each of which includes two different structures, in

the flexibility-index range 74.2 K to 150 K. For this particular problem 150 K flexibility index is the problem limit (dictated by the problem definition given in Table 1).

Table 1: Definition of the example HEN problem (Yee and Grossmann, 1991).

Stream	HCF (kW/K)	Inlet T (K)	Outlet T (K)	h (kW/m^{2}K)	Cost ($/kW-yr)
H1	10	650	370	1	
H2	20	590	370	1	
C1	15	650	650	1	
C2	13	500	500	1	
Steam		680	680	5	85
Water		300	320	1	15
Exchanger Cost					5500+150A

Using the second stage of the methodology proposed in this work (Eqs P2.1-P2.28), for the structures (A, B, C2, D2) (Figure 2), the minimum total cost for different δ_{RD} values is investigated. Bypass placements to the structures are done so that every exchanger in the network has only one bypass and this bypass is on the stream with larger heat-capacity-flowrate. The total costs of Structures A, B, C2, and D2 are shown in Figure 3. The following conclusions can be drawn from Figure 3. i) Although the (structural) flexibility index of Structure A is 37.0 K, within the retrofit-design-flexibility-index range of 35.7 K to 37.0 K, Structure B is less expensive and more flexible (more robust to input-temperature disturbances) than Structure A. Same thing can be said when comparing C2 with B in the range 72.5 K to 74.2 K, and when comparing D2 with C2 in the range 85 K to 90 K. ii) The total costs of the structures A, B, and C2 do not change with the retrofit-design flexibility index from 0 up to structure-dependent specific values (up to 30.7 K for Structure A, up to 61.0 K for Structure B, and up to 120.0 K for Structure C2). These values are the design flexibility indexes (δ_D) of these structures with the specified bypass locations ($\delta_D = \delta_{RD}$) Hence, it is not necessary to do retrofit design (area adjustments) if the desired flexibility is in those ranges for these structures. Table 2 summarizes the results obtained from the proposed methodology that may be used as selection criteria among the HEN structures.

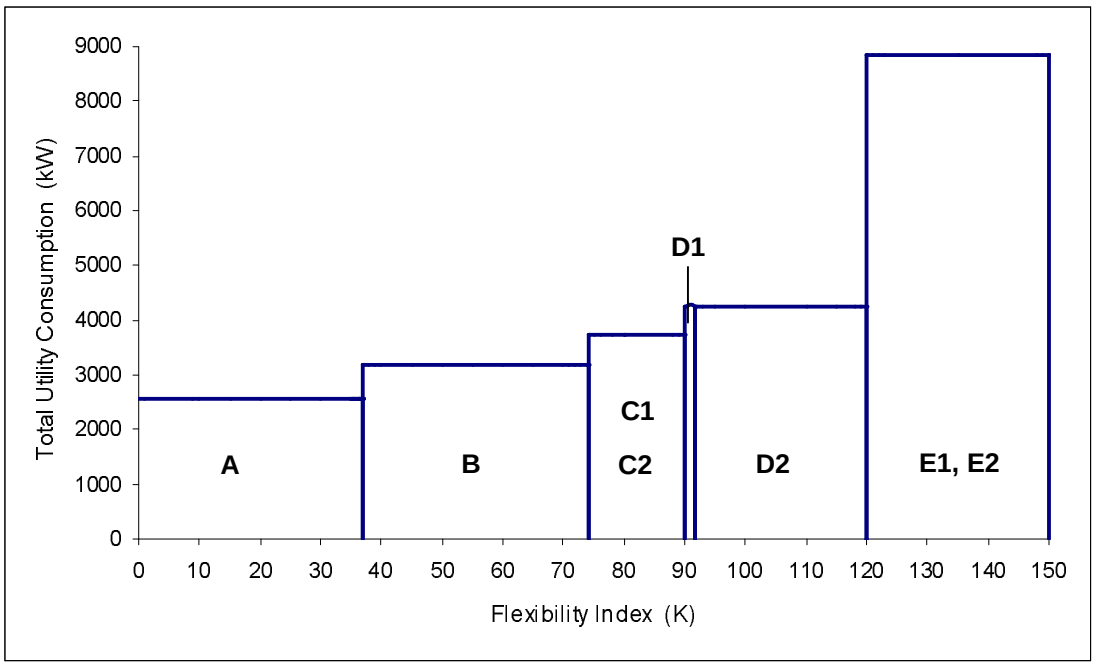


Figure 1. Utility consumption at the nominal operating condition versus the flexibility index for Structures A through E2.

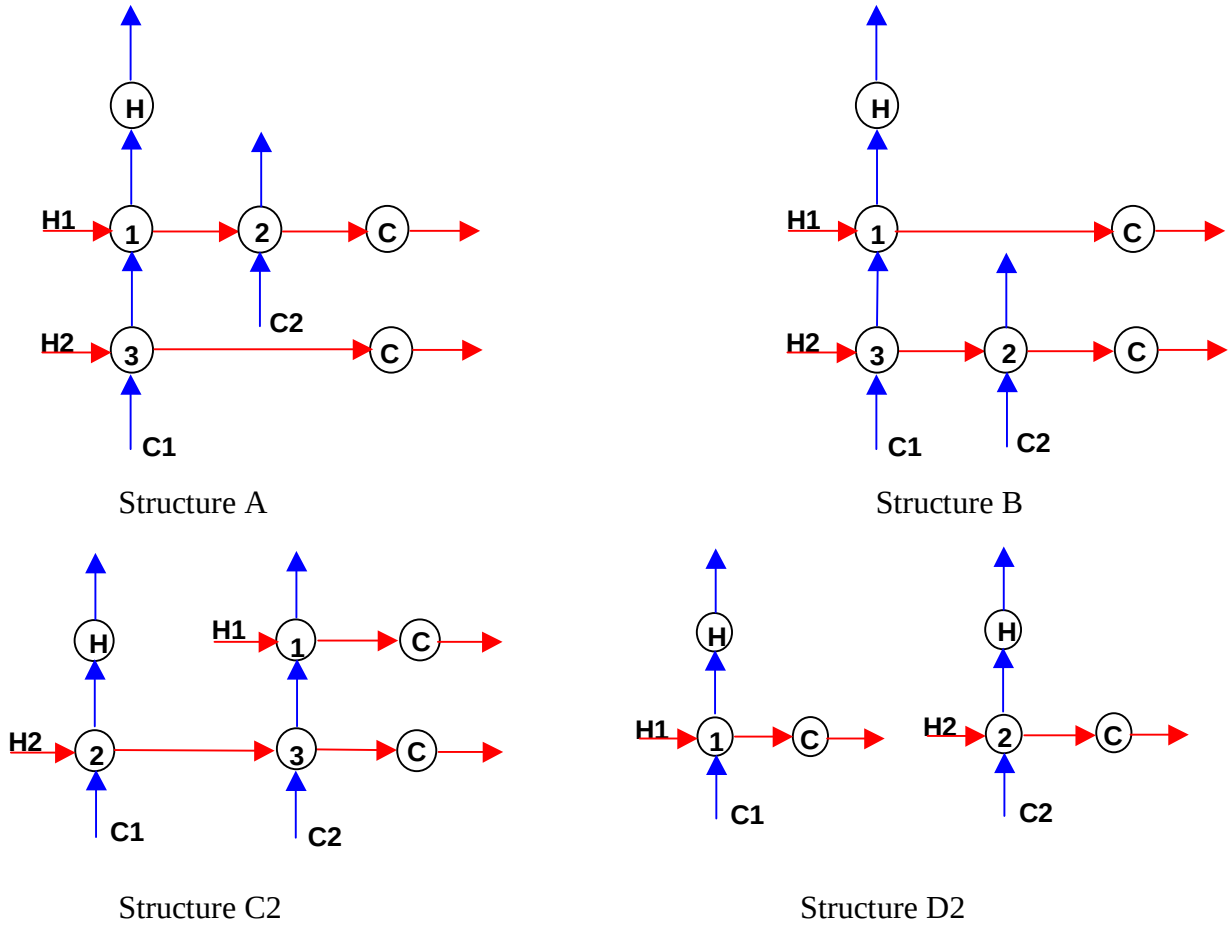


Figure 2. Structures A, B, C2 and D2 generated in Stage 1.

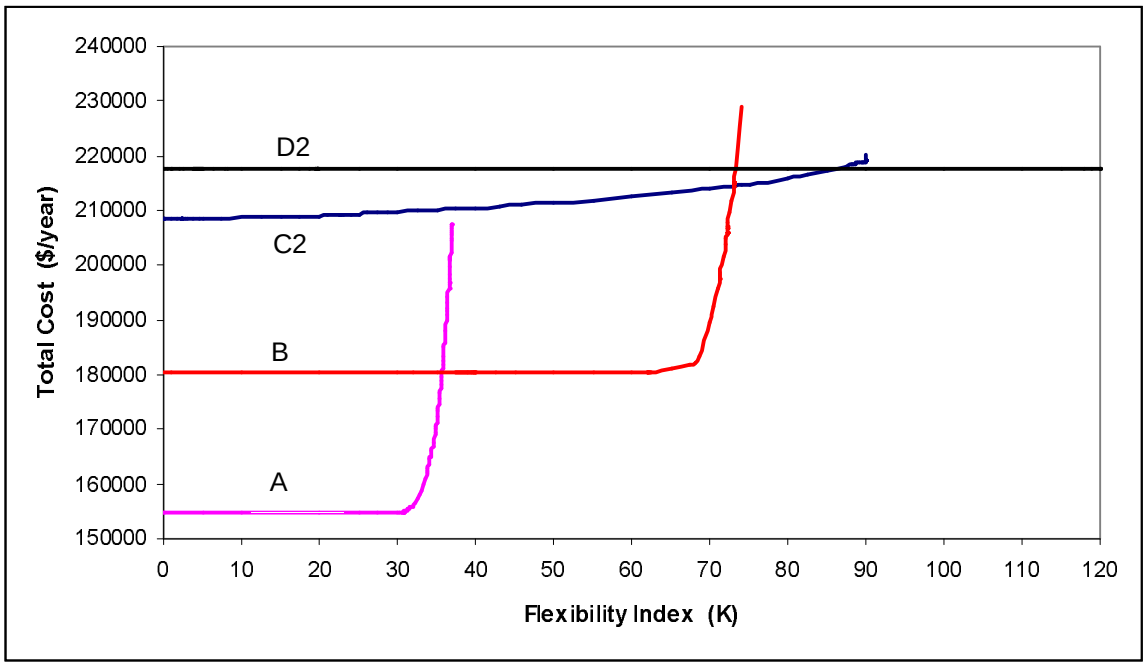


Figure 3: Total Cost of Structures A, B, C2 and D2 against the flexibility index.

Table 2: Flexibilities of Structures A, B, C2 and D2.

	energy optimal solution (stage 1)		total-cost solution (stage 2)
	δ^* (K)	δ_D (K)	δ_{RD} (K)
Structure A	0.0 - 37.0	30.7	0.0 - 35.7
Structure B	37.0 - 74.2	61.0	35.7 - 72.5
Structure C2	74.2 - 90.0	0.0	72.5 - 85.0
Structure D2	90.0 - 120.0	120.0	85.0 - 120.0

Conclusions

In this work, a two-stage methodology was proposed to design a heat exchanger network that has desired operability/controllability range at the least cost. In the first stage, energy-optimal HEN structures satisfying desired flexibility-index were generated. The proposed MILP formulation in this stage may result one or more maximum-energy-recovery HEN structures all operable at the same specified operation range defined by the flexibility-index. In the second stage of the methodology, the energy-optimal structures were retrofitted via a vertex-based, one-stage nonlinear programming formulation to obtain a total-cost-optimal solution. Applying the methodology to an example HEN problem, it was also shown that the operability range of the HEN, designed without considering its flexibility (the placement of bypasses after the design) gives an operating range smaller than its structural flexibility. The cost of increasing the design flexibility of a structure up to its structural flexibility is the increase in the total area (investment cost) of the network. It was also shown in the example that it was possible to find less expensive networks which are not energy-optimal in the desired flexibility range, however, with lower total-cost, and robust to input-temperature disturbances.

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Nomenclature

A	area of exchangers
C	investment or operating cost coefficients
d	vector of design variables
g	vector of inequality constraints
h	vector of equality constraints
h	individual heat transfer coefficients for streams
n	vertex-direction identifier (subscript or superscript)
N	total number of vertices
N_C	number of cold streams
N_H	number of hot streams
N_S	number of superstructure stages
NC	number of coolers in the network
NCB	number of cold-side bypasses in the network
NCS	number of cold streams in the network
NE	number of exchangers in the network
NH	number of heaters in the network
NHB	number of hot-side bypasses in the network
NHS	number of hot streams in the network
NHT	number of hard targets
NS	number of source streams entering the network
NST	number of soft targets
NX	number of stream splits
Q	heat loads
p	vector of fixed parameters
$\mathbf{r}_n$	operating-point identifiers as N combinations of –1 and +1
R	region describing the scaled hyperrectangle
ST	index set of superstructure stages
T	temperature
u	bypass fractions
u	vector of free/control variables
U	overall heat transfer coefficient
x	split fractions
x	vector of state variables
w	heat capacity flowrates
z	vector of binary variables

α, β	investment cost coefficients for heat exchangers
δ	flexibility (scalar)
δ^*	flexibility index (scalar)
δ_D	design flexibility index (scalar)
δ_{RD}	retrofit design flexibility index (scalar)
Φ	objective function
θ	vector of uncertain parameters
$\Delta\theta^-$, $\Delta\theta^+$	scaled deviations from θ^o
ΔT_{min}	minimum approach temperature

Subscripts

ce	cold exit of exchanger
he	hot exit of exchanger
ci	cold input of exchanger
hi	hot input of exchanger
cf	cold final after bypass
hf	hot final after bypass
lm	logarithmic mean
i	i^{th} disturbance direction
j	j^{th} exchanger
k	k^{th} cooler
l	l^{th} heater
C	cooler
CU	cold utility
D	design
E	exchanger
H	heater
HU	hot utility
RD	retrofit design

Superscripts

i	disturbance direction
in	inlet
min	minimum
max	maximum
o	value at nominal operating condition
out	outlet
t	target

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