Multiperiod Synthesis of Minimum-Utility Heat-Exchanger Networks with Period-Specific Structural Flexibilities through Superstructure-Based Simultaneous MILP Formulation

A.E.S. Konukman[*], U. Akman[+]

[*] Dept. Energy Systems Eng., Gebze Institute of Technology, Gebze 41400, Kocaeli, Turkey
Tel: 90 (262) 6538497-1062, Fax: 90 (262) 6538490, E-mail: konukman@penta.gyte.edu.tr

[+] Dept. Chem. Eng., Bogazici University, Bebek 80815, Istanbul, Turkey
Tel: +90 (212) 2631540-1867, Fax: +90 (212) 2872460, E-mail: akman@boun.edu.tr

Abstract – A superstructure-based MILP (Mixed Integer Linear Programing) formulation for simultaneous flexibility targeting and synthesis of multiperiod minimum-utility Heat-Exchanger Networks (HENs) is presented. The formulation is solved for the special case where period-specific structural flexibilities are explored on the basis of vertices of polyhedral regions of uncertainties at all periods of operation. As an example, a two-period HEN problem is solved for increasing structural-flexibility targets, and the resulting HEN structures that are operable in both periods with desired flexibilities are analyzed in terms of structural flexibility indexes, structural grouping, and total utility consumption levels.

Introduction

The synthesis, design, and optimization of multiperiod process plants, where operational characteristics, production demands, and costs vary from period to period due to changing market conditions, are receiving significant focus (Sahinidis et al., 1989; Paules and Floudas, 1992; Varvarezos et al., 1992; Papalexandri and Pistikopoulos, 1994; Varvarezos et al., 1994; Varvarezos et al., 1995; Dedopoulos and Shah, 1996; Iyer and Grossmann, 1997; Bhatia and Biegler, 1999; Rooney and Biegler, 1999; Van den Heever and Grossmann, 1999; Bok et al., 2000). Although the synthesis, design, and optimization of multiperiod processes have received considerable attention, the works that consider period-dependent flexibility (ability to tolerate uncertainties at all periods of operation) of a multiperiod process are rare. In most of the works that deal with the synthesis, design, and optimization of flexible processes, the "periods" are not "real operation periods" but discrete (critical) points in uncertainty spaces on which discretized form of a stochastic optimization problem is solved. In other words, the periods are formed by/for the numerical algorithms so that the continuous stochastic optimization problem, which is very difficult to solve, is simplified.

Multiperiod-optimization problems typically involve cost minimization or profit maximization over all periods of operation. Generally, discrete variables represent period-specific discrete decisions such as whether or not to invest in or to operate a process/equipment in a given period. Decomposition of a multiperiod-optimization problem, for the solution of periods independently, is usually prohibited due to linking decision variables and/or constraints that are common to all periods.

In this work, an optimization model for simultaneous period-specific-flexibility targeting and synthesis of multiperiod HENs is presented. The model is solved for a superstructure-based

multiperiod synthesis of minimum-utility flexible HEN problem for the special case where the period-specific flexibilities are explored on the basis of the vertices of the polyhedral regions of uncertainties at all operation periods.

The problem considered in this work can be stated as follows. Givens are a HEN superstructure that should operate over *P* periods and vertex-based flexibility indexes for all periods. The problem involves synthesis of a HEN structure that will operate with minimum utility consumption at the nominal operating conditions of *P* periods and that will posses pre-defined flexibilities to source-temperature disturbances at all periods. The period-specific expected magnitudes of disturbances correspond to the vertices of the polyhedral regions of disturbance space.

The multiperiod synthesis of minimum-utility (maximum-energy-recovery) HEN structures possessing desired structural flexibility indexes is accomplished through a one-stage, non-iterative, superstructure-based simultaneous MILP formulation that achieves minimum-utility-structure synthesis and flexibility targeting for all *P* periods simultaneously in a single MILP solution. The formulation assumes that the multiperiod feasible region defined by the reduced inequality constraints is convex or one-dimensional-quasi-convex in uncertain parameters (considered to be the source-stream temperatures in this work), so that the solution can be explored on the vertices of the multiperiod polyhedral region of uncertainty.

The basis of the formulation is the superstructure-based simultaneous-MINLP formulation developed by Yee and Grossmann (1990a, 1990b) as a solution to the standard HEN-synthesis problem. The original MINLP formulation (a nonlinear total-cost function as objective and a set of linear constraints) is based on a stagewise superstructure representation that considers potential exchanges between any pair of hot and cold streams within a stage via binary decision variables. In this work, the model of Yee and Grossmann is reformulated as MILP and the set of linear constraints is augmented for *P* periods and for all vertices in these periods such that the resulting structure satisfies expected deviations in source-stream temperatures in every combination of disturbance directions (vertex directions) at all periods of operation. Source-stream temperatures are perturbed for each vertex by a period-specific scalar; the desired flexibility targets for the periods. Thus, satisfaction of the linear constraints is required for each vertex as well as for the nominal point at all periods. The target temperatures for all periods, specified by the multiperiod HEN problem, are required to be matched exactly at each vertex and for each period by variable (vertex- and period-dependent) heater/cooler loads. The objective function is the minimization of the weighted sum of the total utility consumptions at the nominal operating points of all periods. Thus, the whole problem becomes MILP for which the superstructure-dependent global solution (general vertex solution) is obtained with the software package GAMS/XA. Since, in this work, the potential process and utility exchangers are required to exist at all periods, the binary variables denoting potential stream matches are common to all periods and thus are the linking variables between the periods.

An example HEN problem is solved for two periods for different structural-flexibility targets, and the resulting HEN structures that are operable in both periods with desired flexibilities are analyzed in terms of flexibility indexes, structural grouping, and utility-consumption levels.

General Formulation

The general Mixed-Integer-Programming (MIP) formulation for a superstructure-based simultaneous multiperiod HEN synthesis/design and flexibility targeting can be stated as follows:

$$\min_{\mathbf{d},\mathbf{x}_p,\mathbf{u}_p,\mathbf{z}} \Phi\left(\mathbf{d},\mathbf{x}_p,\mathbf{u}_p,\mathbf{z},\mathbf{p}_p,\boldsymbol{\theta}_p\right) \qquad p \in P \qquad (1)$$

$$\text{s.t.} \quad \mathbf{h}_p\left(\mathbf{d},\mathbf{x}_p,\mathbf{u}_p,\mathbf{z},\mathbf{p}_p,\boldsymbol{\theta}_p\right) = \mathbf{0} \qquad p \in P \qquad (2)$$

$$\mathbf{g}_p\left(\mathbf{d},\mathbf{x}_p,\mathbf{u}_p,\mathbf{z},\mathbf{p}_p,\boldsymbol{\theta}_p\right) \le \mathbf{0} \qquad p \in P \qquad (3)$$

$$\theta_p \in R\left(\delta_p\right) = \left\{\theta_p \,\middle|\, \left(\theta_p^o - \delta_p \Delta\theta_p^-\right) \le \theta_p \le \left(\theta_p^o + \delta_p \Delta\theta_p^+\right)\right\} \qquad p \in P \qquad (4)$$

where $\mathbf{d}$ is the vector of period-independent design variables (exchanger areas), $\mathbf{x}_p$ is the vector of state variables (outlet temperatures) at each period, $\mathbf{z}$ is the vector of period-independent binary variables (denoting potential existence/absence of exchangers embedded in the superstructure that is common to each period), $\mathbf{u}_p$ is the vector of control/free variables (heater/cooler loads) at each period, $\mathbf{p}_p$ is the vector of fixed parameters (heat-transfer coefficients) at each period, and $\boldsymbol{\theta}_p$ is the vector of uncertain parameters at each period (in this work, source-stream temperatures). Eq.1 is the objective function and can be formulated as weighted total annualized cost or total utility consumption at each period. For a synthesis problem, the vector $\mathbf{d}$ is dropped from the set of decision variables. Eqs.2-3 are the vector of equality and inequality constraints for each period. If the vector of states, $\mathbf{x}_p$, can be eliminated using Eq.2, then Eq.3 can be written as the vector of reduced inequalities. Eq.4 describes the range of the uncertain parameters as deviations from period-dependent nominal values, $\boldsymbol{\theta}_p^o$, where $\Delta\boldsymbol{\theta}_p^-$ and $\Delta\boldsymbol{\theta}_p^+$ are the scaled deviations from $\boldsymbol{\theta}_p^o$, and the non-negative scalar, δ_p, is the targeted flexibility for period *p*. The region $R(\delta_p)$ describes the scaled hyperrectangle which is required to be within the feasible region defined by Eqs.2-3. For a fixed HEN topology and design (known $\mathbf{z}$ and $\mathbf{d}$), the flexibility index, δ_p^*, defined by Swaney and Grossmann (1985) is the maximum of scalar δ_p. This problem (Eqs.1-4) is difficult to solve directly since, due to Eq.4, it involves a max-min-max constraint that leads to a nondifferentiable global-optimization problem (Halemane and Grossmann, 1983). The problem can be solved indirectly with a two-stage procedure; as iterations between a multiperiod synthesis problem and a flexibility-analysis step (Papalexandri and Pistikopoulos, 1993), however, this technique requires the solution of large MIPs through, for example, the Generalized Benders Decomposition (Floudas, 1995) in each iteration.

In this work, the multiperiod synthesis of minimum-utility HEN structures satisfying given flexibility target for each period will be accomplished through one-stage, non-iterative, superstructure-based formulation with the supposition that the feasible region defined by the reduced inequality constraints is convex in uncertain parameters. Therefore, the optimal solution can be explored on the basis of the vertices of the polyhedral regions of uncertainties at each period. For this purpose, the general multiperiod vertex-based formulation (Konukman et al., 1999) is described as follows:

$$\min_{\mathbf{x}_p^n,\mathbf{z}^o,\mathbf{u}_p^n} \Phi\left(\mathbf{d}^o,\mathbf{x}_p^n,\mathbf{u}_p^n,\mathbf{z}^o,\mathbf{p}_p,\boldsymbol{\theta}_p^n\right) \qquad n = 0, 1, ..., 2^N \quad p \in P \qquad (5)$$

$$\text{s.t.} \quad \mathbf{h}_p^n\left(\mathbf{d}^o, \mathbf{x}_p^n, \mathbf{z}^o, \mathbf{u}_p^n, \mathbf{p}_p, \boldsymbol{\theta}_p\right) = \mathbf{0} \qquad n = 0, 1, ..., 2^N \quad p \in P \qquad (6)$$

$$\mathbf{g}_p^n\left(\mathbf{d}^o, \mathbf{x}_p^n, \mathbf{z}^o, \mathbf{u}_p^n, \mathbf{p}_p, \boldsymbol{\theta}_p\right) \le \mathbf{0} \qquad n = 0, 1, ..., 2^N \quad p \in P \qquad (7)$$

$$\boldsymbol{\theta}_p - \boldsymbol{\theta}_p^n = \mathbf{0} \qquad n = 0 \quad p \in P \qquad (8)$$

$$\boldsymbol{\theta}_p - \boldsymbol{\theta}_p^n - \delta_p \boldsymbol{\theta}_p^n = \mathbf{0} \qquad n = 1, ..., 2^N \quad p \in P \qquad (9)$$

$$\theta_p^n \in R_o(\delta_p) = \left\{ \theta_p^n \middle| \theta_p^n \text{ is a vertex of } R(\delta_p), n = 1,...,2^N \right\} \qquad p \in P \qquad (10)$$

The vertex formulation, described by Eqs.5-10, is a reformulation of the more general non-vertex formulation, described by Eqs.1-4, using the approach named as "Algorithm I" by Halemane and Grossmann (1983). In the case the feasible region defined by the reduced inequality constraints is convex in uncertain parameters, θ_p, the formulations described by Eqs.1-4 and Eqs.5-10 are identical (Halemane and Grossmann, 1983; Swaney and Grossmann, 1985). For a HEN problem where the uncertain parameters are considered to be the source-stream temperatures only (constant heat-capacity-flowrates, fixed splits and fixed bypass flows), the feasible region defined by the reduced inequality constraints is convex (Floudas, 1995).

Eq.5 is the objective function that is evaluated at the nominal operating points, *n=0*, of all periods. Eqs.6-7 are the augmented forms of the vector of equality and inequality constraints (Eqs.2-3) that are evaluated at the nominal operating points (*n=0*) as well as at each vertex (*n=1,...,2^N*) of all periods simultaneously. The θ_p values in Eqs.6-7 are governed by Eqs.8-9. Eq.9 constrains the vector θ_p to take only 2^N vertex values, θ_p^n (2^N ± directed deviations from the nominal, θ_p^o, values), scaled by the scalar, δ_p, which is the desired flexibility at period *p*. Eq.8 denotes that at *n=0*, δ_p is considered to be zero. The set $R_o(\delta_p)$ describes the vertices of the scaled hyperrectangle which is required to be within the feasible region defined by Eqs.6-7.

As indicated in Eq.5, a unique, and period independent, set of binary decision variables, $\mathbf{z}^o$, controlling the structural modifications, applies to all 2^N+1 operating points (2^N vertex points plus the nominal operating point). This enforces the optimal structure to be reduced from the superstructure to be valid at all 2^N+1 operating points and at all *P* periods. The state variables, $\mathbf{x}_p^n$ (e.g., the exchanger outlet temperatures), and the control/free variables, $\mathbf{u}_p^n$ (e.g., heater/cooler and exchanger heat loads), are the vectors of period-dependent continuous decision variables defined for all 2^N+1 operating points and for *P* periods. In this way, the optimal structure which is unique to all operating points/periods and the optimal values of the operating-point-/period-dependent state and control vectors are obtained simultaneously as the solution of a single MIP problem defined by Eqs.5-10.

Formulation for the Specific HEN Superstructure

The specific form, and the total number of decision variables and constraints of the general vertex-based "simultaneous multiperiod synthesis and flexibility targeting" formulation described by Eqs.5-10 depends on the particular superstructure representation of the process. In this work, Eqs.5-10 are applied to the HEN superstructure proposed by Yee and Grossmann (1990a, 1990b). The specific formulation presented here is based on the simultaneous-MINLP model developed by Yee and Grossmann as a solution to the standard HEN-synthesis problem. The original MINLP formulation of Yee and Grossmann (a nonlinear

total-cost function as objective and a set of linear constraints) is based on a stagewise superstructure representation that considers potential exchanges between any pair of hot and cold streams within a superstructure stage via binary decision variables.

In this work, the model of Yee and Grossmann is reformulated as MILP, by considering the weighted sum of the total utility consumptions at the nominal operating conditions of the periods as the linear objective function. The set of linear constraints is augmented for each vertex and for each period such that the resulting HEN structure satisfies expected deviations, with magnitude δ_p, in source-stream temperatures in every combination of disturbance directions (vertex directions) which are identified by an additional index *n*. In other words, the superstructure is forced to yield a unique minimum-utility structure that is operable at the nominal condition and at each vertex of the feasible region the size of which is determined by the desired flexibility index, at each period *p*. The nominal operating condition of each period at the centeroid of the polyhedral region of uncertainty (operation under no disturbance in source-stream temperatures) is identified by *n=0*. For a HEN problem with N_H hot streams and N_C cold streams, there are $N=2^{(N_H+N_C)}$ vertices (+ and – directed simultaneous disturbance directions) besides the nominal (no disturbance) operating point. Overall heat balances for each stream, heat balances at each stage of the superstructure, assignment of superstructure inlet temperatures, equations for the feasibility of temperatures, hot-/cold-utility-load equations, logical constraints, and approach-temperature equations forming the set of linear constraints in the original formulation of Yee and Grossmann are all written for each of the $2^{(N_H+N_C)}$ vertices as well as for the nominal operating point for all *P* periods. Source-stream temperatures are perturbed for each vertex by a fixed scalar, δ_p, which is the targeted flexibility for period *p*. Thus, satisfaction of the linear constraints is required for each vertex and for the nominal operating points of all periods. The target temperatures of each period, specified by the HEN problem, are required to be matched exactly for each vertex by variable (vertex-/period-dependent) heater/cooler and exchanger heat loads. The objective function is the minimization of the weighted sum of the total-utility consumptions of the periods at the nominal operating points; *n=0*. Thus, the whole problem becomes a MILP for which the superstructure-dependent global solution can be obtained. To preserve the linearity of the constraints, the isothermal-mixing assumption of Yee and Grossmann (which is rigorous if the resulting HEN does not involve stream splits but may lead to overestimation of the area cost in the case of stream splits) is retained.

The explicit MILP formulation, depending on the superstructure-based MINLP formulation of Yee and Grossmann, is given by Eqs.(11-36) (Konukman et al., 1999). The index *n* is the vertex identifier for the $2^{(N_H+N_C)}$ vertices, and *n=0*, a non-vertex point, designates the nominal operating condition. The index *p* is the period identifier. *HP* and *CP* are the index sets for the hot and cold process streams. *ST* is the index set for the superstructure stages. The two-stage superstructure, that will be used for an example HEN problem, is shown in Fig 1.

Objective function:

$$\min_{\substack{t_{i,k,n,p},\, t_{j,k,n,p},\\ z_{i,j,k},\, zcu_i,\, zhu_j,\\ q_{i,j,k,n,p},\, qcu_{i,n,p},\, qhu_{j,n,p},\\ dt_{i,j,k,n,p},\, dtcu_{i,n,p},\, dthu_{j,n,p}}} \sum_{p\in P}\alpha_p\left(\sum_{i\in HP} qcu_{i,n=0,p} + \sum_{j\in CP} qhu_{j,n=0,p}\right) \quad k\in ST,\ n=0,1,...,N \quad (11)$$

The objective (Eq.11) is the minimization of the period *p* weighted sum of cold- (**qcu**) and hot- (**qhu**) utility consumptions at the nominal conditions, *n=0*. The binary decision-variable vectors, **z**, **zcu**, and **zhu** denote potential existence of the process-stream matches, and potential cold- and hot-utility heat exchanges, respectively. They are defined as period-independent global variables, i.e. they are common to the nominal operating condition, *n=0*, to each vertex, and to all *P* periods; since the potential structure is unique for all vertices as well as for the nominal operating conditions of all periods. This global definition of the structure-determining binary decision variables enables the constraints to be satisfied at every vertex as well as at the nominal points over the unique structure by the vertex-/period-dependent continuous decision variables. The vector of continuous decision variables are: **t** (temperatures at the inlet and outlet of each stage in the superstructure for each vertex, for the nominal points, and for all periods), **q** (process-stream heat exchanges at each stage for each vertex, for the nominal points, and for all periods), **qcu** (process/cold-utility heat exchanges at superstructure boundaries for each vertex, for the nominal points, and for all periods), **qhu** (process/hot-utility heat exchanges at superstructure boundaries for each vertex, for the nominal points, and for all periods), **dt** (approach temperatures for process-stream matches at stage boundaries for each vertex, for the nominal points, and for all periods), **dtcu** (approach temperatures for process/cold-utility matches at superstructure boundaries for each vertex, for the nominal points, and for all periods), and **dthu** (approach temperatures for process/hot-utility matches at superstructure boundaries for each vertex, for the nominal point, and for all periods).

Overall heat balance for each stream, for each vertex, for the nominal points, and for all periods:

$$\left(T^{in}_{i,n,p} - T^{out}_{i,p}\right)F_{i,p} = \sum_{k\in ST}\sum_{j\in CP} q_{i,j,k,n,p} + qcu_{i,n,p} \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (12)$$

$$\left(T^{out}_{j,p} - T^{in}_{j,n,p}\right)F_{j,p} = \sum_{k\in ST}\sum_{i\in HP} q_{i,j,k,n,p} + qhu_{j,n,p} \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (13)$$

The overall heat-balance constraints (Eqs.12-13) imply that the overall heat-exchange requirement of each stream must be equal to the sum of the heat exchanged with other process streams and with the utility stream, at each vertex, at the nominal operating point, and at all periods.

Heat balance at each stage, for each vertex, for the nominal points, and for all periods:

$$\left(t_{i,k,n,p} - t_{i,k+1,n,p}\right)F_{i,p} = \sum_{j\in CP} q_{i,j,k,n,p} \qquad k\in ST,\ i\in HP,\ n=0,1,...,N,\ p\in P \qquad (14)$$

$$\left(t_{j,k,n,p} - t_{j,k+1,n,p}\right)F_{j,p} = \sum_{i\in HP} q_{i,j,k,n,p} \qquad k\in ST,\ j\in CP,\ n=0,1,...,N,\ p\in P \qquad (15)$$

The stage heat-balance constraints (Eqs.14-15) are needed to determine the stage temperatures, at each vertex, at the nominal operating point, and at all periods.

Assignment of inlet temperatures, for each vertex, and for target flexibilities, δ_p, for all periods:

$$T^{in}_{i,n,p} = \left(T^{in,o}_{i,p} + r_{i,n}\,\delta_p\right) = t_{i,1,n,p} \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (16)$$

$$T^{in}_{j,n,p} = \left(T^{in,o}_{j,p} + r_{j,n}\,\delta_p\right) = t_{j,ST+1,n,p} \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (17)$$

The inlet temperatures of the process streams are assigned to the inlet temperatures of the superstructure through Eqs.16-17 as dependent on the vertices of the polyhedral uncertainty region, the size of which is defined by the scalar targeted period flexibilities, δ_p, that act as scale factors. The set $\mathbf{r}_n$ ($n\neq 0$) is the vertex (disturbance direction) identifier and takes the values of $N= 2^{(N_H+N_C)}$ combinations of +1 and −1. For the nominal operating points of the periods (centroids of the polyhedral regions) the values of $\mathbf{r}_o$ are zero for all $i\in HP$ and $j\in CP$, and at this point, the superstructure inlet temperatures are assigned the nominal source-stream temperatures corresponding to period *p*, $\mathbf{T}_p^{in,o}$.

Feasibility of temperatures, for each vertex, for the nominal points, and for all periods:

$$t_{i,k,n,p} \geq t_{i,k+1,n,p} \qquad k\in ST,\ i\in HP,\ n=0,1,...,N,\ p\in P \qquad (18)$$

$$t_{j,k,n,p} \geq t_{j,k+1,n,p} \qquad k\in ST,\ j\in CP,\ n=0,1,...,N,\ p\in P \qquad (19)$$

$$T_{i,p}^{out} \leq t_{i,ST+1,n,p} \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (20)$$

$$T_{j,p}^{out} \geq t_{j,1,n,p} \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (21)$$

Eqs.18-19 constrain the stage temperatures to be monotonically decreasing at each vertex and at the nominal operating points of the periods. Eqs.20-21 bound the superstructure outlet temperatures by the fixed outlet temperatures of the process streams of each period, $\mathbf{T}_p^{out}$. If these last two constraints are not active at a particular vertex, *n*, or at the nominal operating point, *n=0*, then the stream corresponding to the inactive constraint requires a utility exchanger at that particular vertex, *n*, or at the nominal operating point, *n=0*.

Hot and cold utility loads, for each vertex, for the nominal points, and for all periods:

$$\left(t_{i,ST+1,n,p} - T_{i,p}^{out}\right)F_{i,p} = qcu_{i,n,p} \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (22)$$

$$\left(T_{j,p}^{out} - t_{j,1,n,p}\right)F_{j,p} = qhu_{j,n,p} \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (23)$$

Constraints given by Eqs.22-23 determine the hot (**qhu**) and cold (**qcu**) utility requirements at each vertex and at the nominal operating points. If a particular constraint in the set given by Eqs.20-21 is active then the corresponding utility requirement becomes zero and the reduced structure does not contain a utility exchanger for the particular process stream at the particular operating point.

Logical constraints, for each vertex, for the nominal points, and for all periods:

$$q_{i,j,k,n,p} - \Omega_{i,j,n,p} z_{i,j,k} \leq 0 \qquad k\in ST,\ i\in HP,\ j\in CP,\ n=0,1,...,N,\ p\in P \qquad (24)$$

$$qcu_{i,n,p} - \Omega_{i,n,p} zcu_i \leq 0 \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (25)$$

$$qhu_{j,n,p} - \Omega_{j,n,p} zhu_j \leq 0 \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (26)$$

Logical constraints (Eqs.24-26) involving the binary variables determine the existence or absence of process, cold-utility, and hot-utility matches. The binary decision-variable vectors, **z**, **zcu**, and **zhu**, are common to the nominal operating conditions *n=0* and to each vertex of all periods. This is because the unique potential structure must be operable at all vertices and at the nominal conditions of all periods, and with this global structure, the constraints should be satisfied at every vertex as well as at the nominal points by the vertex-/period-dependent continuous decision variables. Upper bounds, $\Omega_{i,j,n,p}$, in these logical constraints can safely be

set to the smallest heat content of the two streams involved in the match. Heat-content calculation over the superstructure must be based on the source- and target-stream temperatures at the corresponding operating points, *n=0,...,N*, of the periods since the source-stream temperatures are perturbed for each vertex by the scalar period-specific target flexibilities as described by Eqs.16-17.

Approach temperatures, for each vertex, for the nominal points, and for all periods:

$$dt_{i,j,k,n,p} \le t_{i,k,n,p} - t_{j,k,n,p} + \Gamma_{i,j,n,p}\left(1 - z_{i,j,k}\right) \qquad k\in ST,\ i\in HP,\ j\in CP,\ n=0,1,...,N,\ p\in P \qquad (27)$$

$$dt_{i,j,k+1,n,p} \le t_{i,k+1,n,p} - t_{j,k+1,n,p} + \Gamma_{i,j,n,p}\left(1 - z_{i,j,k}\right) \qquad k\in ST,\ i\in HP,\ j\in CP,\ n=0,1,...,N,\ p\in P \qquad (28)$$

$$dtcu_{i,n,p} \le t_{i,ST+1,n,p} - T^{out}_{CU,p} + \Gamma_{i,n,p}\left(1 - zcu_i\right) \qquad i\in HP,\ n=0,1,...,N,\ p\in P \qquad (29)$$

$$dthu_{j,n,p} \le T^{out}_{HU,p} - t_{j,1,n,p} + \Gamma_{j,n,p}\left(1 - zhu_j\right) \qquad j\in CP,\ n=0,1,...,N,\ p\in P \qquad (30)$$

The approach-temperature constraints (Eqs.27-30) ensure positive driving forces for the potential process and utility exchangers at the nominal operating points of the periods and at each vertex. The binary variables in these constraints ensure that only the non-negative driving forces exist for potential matches (binary variables are equal to one). If there is no match (binary variables are equal to zero) in any stage of the superstructure at any operating point, then the upper bounds on the driving forces are safely set to the largest of the possible driving forces through proper definition of the upper-bound parameters $\Gamma_{i,j,n,p}$, which are calculated based on the period-dependent inlet and outlet temperatures of the superstructure. The upper-bound parameters must be calculated not only at the nominal operating point but at the corresponding operating points, *n=0,...,N*, of the periods since the source temperatures are perturbed for each vertex by period-specific target flexibilities as described by Eqs.16-17.

Approach-temperature bounds, for each vertex, for the nominal points, and for all periods:

$$dt_{i,j,k,n=0,p} \ge \Delta T_{min} \qquad k\in ST,\ i\in HP,\ j\in CP,\ n=0,\ p\in P \qquad (31)$$

$$dtcu_{i,n=0,p} \ge \Delta T_{min} \qquad i\in HP,\ n=0,\ p\in P \qquad (32)$$

$$dthu_{j,n=0,p} \ge \Delta T_{min} \qquad j\in CP,\ n=0,\ p\in P \qquad (33)$$

$$dt_{i,j,k,n\ne 0,p} \ge 0 \qquad k\in ST,\ i\in HP,\ j\in CP,\ n=1,...,N,\ p\in P \qquad (34)$$

$$dtcu_{i,n=0,p} \ge 0 \qquad i\in HP,\ n=0,\ p\in P \qquad (35)$$

$$dthu_{j,n=0,p} \ge 0 \qquad j\in CP,\ n=0,\ p\in P \qquad (36)$$

The formulation presented here is not for the design (area targeting) but for the synthesis of HENs. However, the approach temperatures for the process and utility streams at the nominal operating point (Eqs.31-33) are bounded by the minimum-approach temperature, ΔT_{min}, considering the possible design of the synthesized structure at the nominal operating point. On the other hand, the approach temperatures for the process and utility streams at the vertices (Eqs.34-36) are required to be positive since the vertex operating points are not considered as possible design points.

The maximum total number of heat-exchange units, NEU_{max}, can be limited (possibly as period dependent) by adding the following constraint.

$$\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} z_{i,j,k} + \sum_{i\in HP} zcu_i + \sum_{j\in CP} zhu_j \le NEU_{max} \qquad p\in P \qquad (37)$$

Since the binary decision variables are global variables, this constraint (Eq.37) is valid at every vertex as well as at the nominal operating points of all periods. Constraining the structure to be reduced from the superstructure in this way, affects the flexibility that should be expected from the reduced structure. However, this constraint generally decreases the solution time and also decreases the chance of getting multiple solutions (same objective function value for the same desired flexibility, but with different sets of binary variables).

The stream splitting may be prevented by constraining the maximum number of matches for each stream at each stage by adding the following constraints:

$$\sum_{j\in CP} z_{i,j,k} \le 1 \qquad k\in ST,\ i\in HP,\ p\in P \qquad (38)$$

$$\sum_{i\in HP} z_{i,j,k} \le 1 \qquad k\in ST,\ j\in CP,\ p\in P \qquad (39)$$

The inclusion of these constraints completely eliminates the isothermal-mixing assumption that is inherent in the formulation of Yee and Grossmann in case of stream splits. Since the binary decision variables are global variables, these constraints (Eq.38 and Eq.39) are valid at every vertex as well as at the nominal operating points of all periods. Constraining the structure to be reduced from the superstructure in this way, affects the flexibility that should be expected from the reduced structure. Similarly, the presence of these constraints generally decreases the solution time and also decreases the chance of getting multiple solutions (same objective function value for the same desired flexibility, but with different reduced structure).

Significance

The superstructure-based, simultaneous-MILP formulation presented in this work synthesizes only the structure for a minimum-utility multiperiod HEN possessing a desired vertex-/period-based *structural flexibility*, δ_p. If the design (determination of process/utility exchanger areas) of the synthesized multiperiod HEN is done, e.g. based on the temperatures corresponding to the nominal operating condition of any of the periods, there is no guarantee that this design will show the same structural flexibility index, δ^*_p, at all operating conditions corresponding to vertices of that period. Actually, if the synthesized structure does not have utility exchangers and the target temperatures are considered to be hard targets, then the *design flexibility index*, $\delta^*_{D,p}$, may be zero. On the other hand, if the retrofit design (placement of bypass streams, possibly with non-zero nominal flows, and re-determination of process/utility exchanger areas) of the HEN is performed, then the *retrofit-design flexibility index*, $\delta^*_{RD,p}$, may be at most as large as the structural flexibility index (if the upper bound for bypass fractions is considered as unity). The ordering of the structural, design, and retrofit-design flexibility indexes can be summarized as:

$$\delta^{*}_{D,p} \le \delta^{*}_{RD,p} \le \delta^{*}_{p} \qquad p \in P \qquad (40)$$

Therefore, the significance of the simultaneous flexibility targeting and synthesis of minimum-utility multiperiod HEN structures is that the flexibility index, δ^{*}_{p}, of the reduced structure provides an upper bound for the flexibility that should be expected at most from the design or retrofit design of the structure for period *p*. Moreover, the value of the objective function, the minimum of total utility consumption, provides a lower bound for the utility consumption level that should be expected at most from the design or retrofit design of the structure aiming at the structural flexibility δ_p for period *p* (since the flexibility lost due to design may be gained at the expense of increasing the utility consumption).

The heat loads for the process/utility matches corresponding to the nominal operating points of the periods and to all vertices are the continuous decision variables and their optimal values are obtained simultaneously as the solution of the MILP formulation. This information on vertex-/period-dependent variations in the heat loads for the matches may reveal possible locations of bypass streams in an ensuing (retrofit) design stage. It may be advantageous to consider bypasses for those matches the heat-load variations of which vary the most from vertex/period to vertex/period. In this way, the bypasses may be placed at the most sensitive locations in the structure.

Example and Discussion

In this section, the basic features of the proposed non-iterative, superstructure-based, simultaneous-MILP formulation for the simultaneous flexibility targeting and synthesis of multiperiod minimum-utility HEN structures are illustrated with an example HEN problem with two periods.

Table 1 gives the supply and target temperatures for the process and utility streams, and the heat-capacity-flowrates of the process streams, for the two periods. Since there are two hot and two cold streams ($N_H=2$, $N_C=2$) in each period, the minimum number of superstructure stages, N_S, is fixed at $max\{N_H,N_C\}=2$ (Yee and Grossmann, 1990a, 1990b). Fig. 1 shows the two-stage superstructure of Yee and Grossmann. With two hot and two cold streams, for each period, the total number of vertices is $N=2^{(N_H+H_C)}=16$. The values of the identifier $\mathbf{r}_n$ (Eqs.16-17) for the nominal operating points and for all the vertices are listed in Table 2.

Table 1. Data for the Two-Period HEN Problem.

Period p = 1				**Period p = 2**			
Stream	F (kW/K)	T^{in} (K)	T^{out} (K)	Stream	F (kW/K)	T^{in} (K)	T^{out} (K)
HOT-1	10	650	370	HOT-1	10	580	360
HOT-2	20	590	370	HOT-2	15	480	320
COLD-1	15	410	650	COLD-1	13	320	460
COLD-2	13	350	500	COLD-2	20	420	520
Hot Utility		680	680	Hot Utility		680	680
Cold Utility		300	320	Cold Utility		300	320

The total number of matches was constrained, for both periods, to be less than or equal to 6 via the addition of the inequality constraint given by Eq.37 with $NEU_{max}=6$. The MILP formulation (Eqs.11-39) was solved with GAMS/XA (Brooke et al., 1992) for the fixed values of the desired flexibility target, δ_p (± directed deviations in source-stream temperatures),

ranging from δ=0K to δ=150K for period *p*=1 (150K is the maximum value of δ_p that can be tolerated for the period specifications), from δ=0K to δ=100K for period *p*=2 (100K is the maximum value of δ_p that can be tolerated for the period specifications), and from δ=0K to δ=100K as multiperiod *P*=2 problem (100K is the maximum value of δ_p that can be tolerated for the multiperiod specifications). In this work, the same desired flexibility target, δ_p was used for both periods in the multiperiod case; meaning that the synthesized multiperiod HEN structure was required to have an identical structural flexibility at both periods of operation.

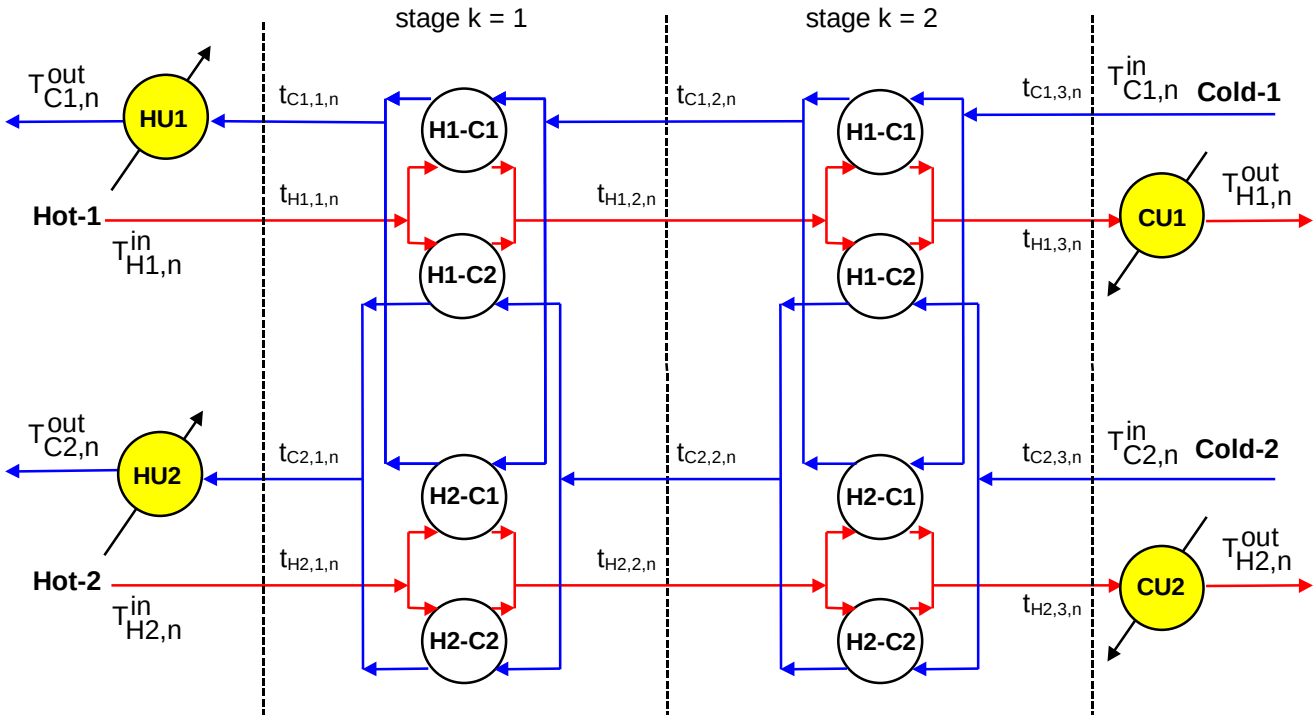
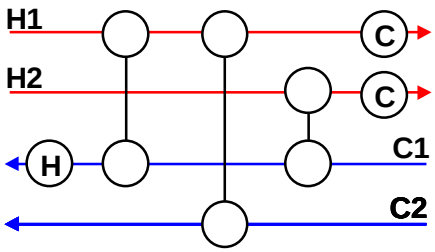


Figure 1. The two-stage superstructure of Yee and Grossmann.

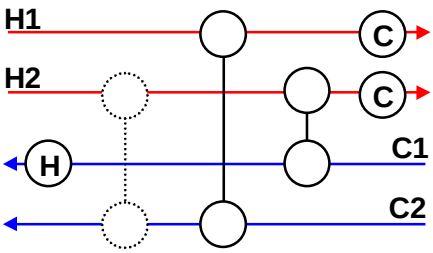
By scanning the possible MILP solutions in the feasible flexibility ranges, sets of structure-determining binary variables were obtained. Out of these MILP solutions, sets of binary variables corresponding to unique structures were labelled, in the order of increasing maximum flexibilities (i.e., flexibility indexes, δ^*_p). When the formulation is solved for the first period alone (*p*=1), the structures are P1A, P1B1-B4, P1C1, P1C2, P1D1, P1D2, and the grid diagrams of the structures are shown in Fig. 2 along with their structural flexibility indexes (δ^*).When the formulation is solved for the second period alone (*p*=2), the structures are P2A1, P2A2, P2A3, P2A4, P2A5, P2A6, P2B, and the grid diagrams of the structures are shown in Fig. 3 along with their structural flexibility indexes (δ^*).When the formulation is solved for both periods simultaneously as multiperiod problem (*P*=2), there are only two structures, P2A6 and P2B (identical to the last two structures in Fig. 3 when the second period, p=2, is solved alone). It should be noticed that P1B4 and P2A6 are identical structures, however their flexibility indexes are δ^*=90K and δ^*=80K, respectively.

Table 2. Values of the Identifier $\mathbf{r}_n$ at the Nominal and Vertex Operating Points.

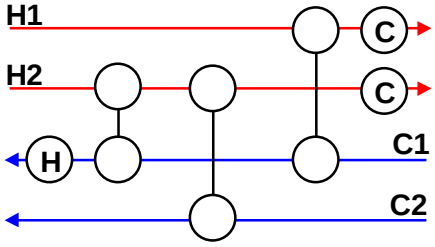
n	$r_{i=1,n}$	$r_{i=2,n}$	$r_{j=1,n}$	$r_{j=2,n}$
0	**0**	**0**	**0**	**0**
1	+	+	+	+
2	+	+	+	–
3	+	+	–	+
4	+	+	–	–
5	+	–	+	+
6	+	–	+	–
7	+	–	–	+
8	+	–	–	–
9	–	+	+	+
10	–	+	+	–
11	–	+	–	+
12	–	+	–	–
13	–	–	+	+
14	–	–	+	-
15	–	–	–	+
16	–	–	–	–



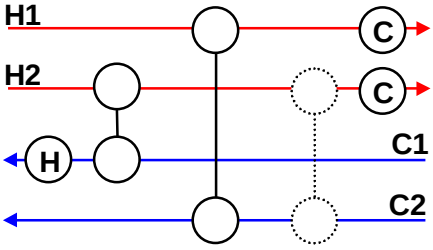
Structure P1A, δ^* = 36.95 K



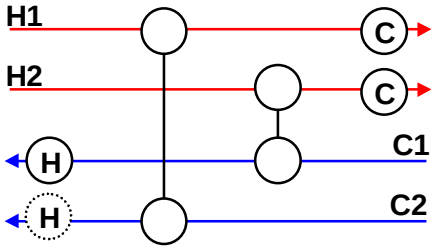
Structure P1B1, δ^* = 70.63 K



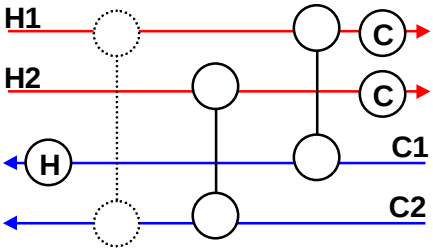
Structure P1B2, δ^* = 74.24 K



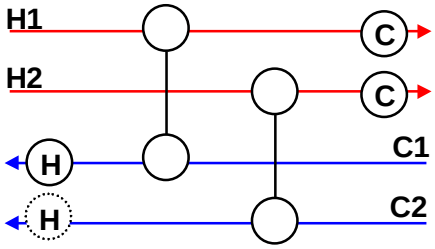
Structure P1B3, δ^* = 90.00 K



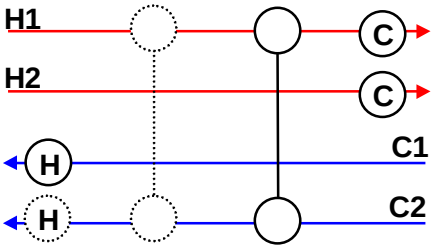
Structure P1B4, δ^* = 90.00 K



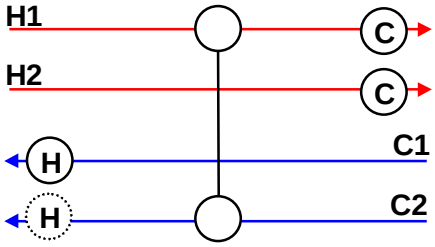
Structure P1C1, δ^* = 91.51 K



Structure P1C2, δ^* = 120.00 K

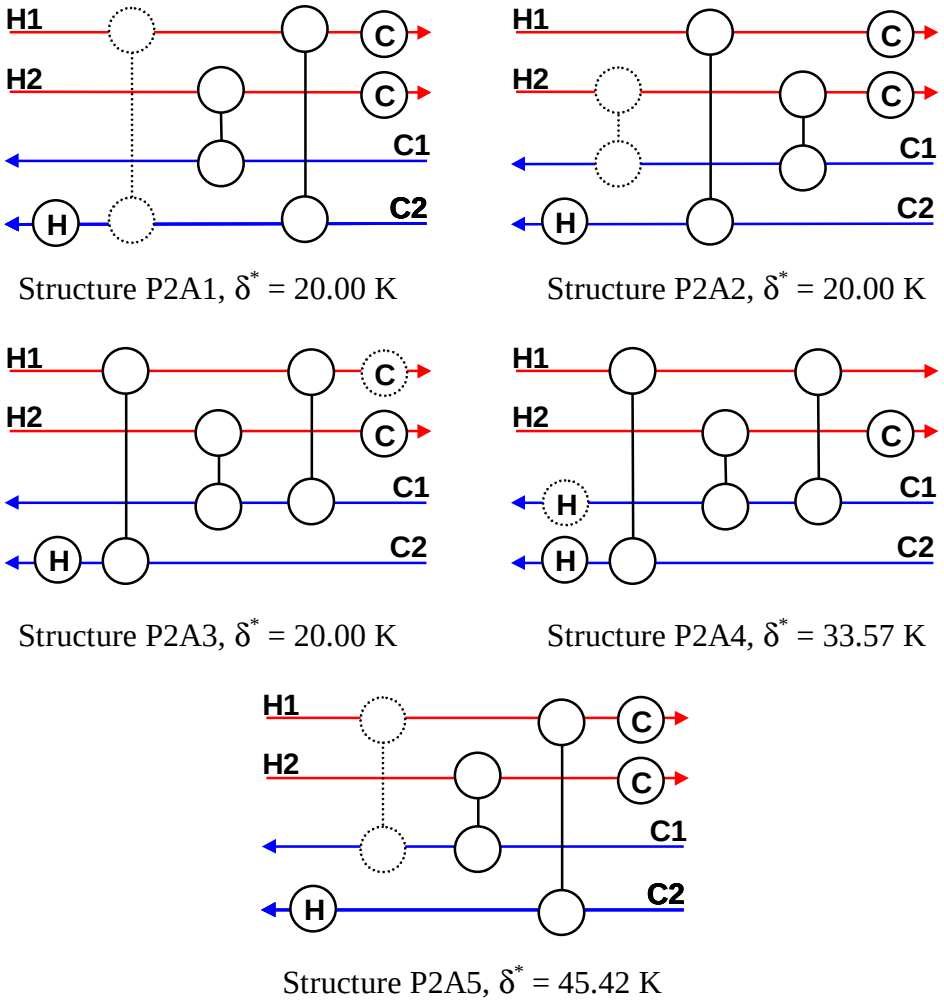


Structure P1D1, δ^* = 150.00 K



Structure P1D2, δ^* = 150.00 K

Figure 2. The grid diagrams of HEN structures P1A through P1D2 obtained for the first period only (the dashed matches are totally bypassed at the nominal operating conditions).



Structure P2A1, $\delta^* = 20.00$ K

Structure P2A2, $\delta^* = 20.00$ K

Structure P2A3, $\delta^* = 20.00$ K

Structure P2A4, $\delta^* = 33.57$ K

Structure P2A5, $\delta^* = 45.42$ K

Structure P2A6, $\delta^* = 80.00$ K

Structure P2B, $\delta^* = 100.00$ K

(also the multiperiod problem structures)

Figure 3. The grid diagrams of HEN structures P2A1 through P2B obtained for the second period only (P2A6 and P2B are also the structures obtained for the multiperiod problem) (the dashed matches are totally bypassed at the nominal operating conditions).

Fig. 4 shows the total-utility-consumption levels corresponding to the nominal operating points ($\sum qcu_{i,n=0} + \sum qhu_{j,n=0}$) of the structures versus the flexibility ranges when the problem is solved for period $p=1$ only, for period $p=2$ only, and for both periods simultaneously as multiperiod problem, $P=2$. Fig. 4 reveals four plateaus of total-utility-requirement levels at the nominal operating point, $n=0$ for period $p=1$, two plateaus of total-utility-requirement levels at the nominal operating point, $n=0$ for period $p=2$, and two plateaus of total-utility-requirement

levels at the nominal operating points, *n=0* when both periods considered simultaneously as multiperiod problem. Each plateau corresponds to one or more (but finite number) of different HEN structures synthesized by solving the MILP for the given value of δ_p.

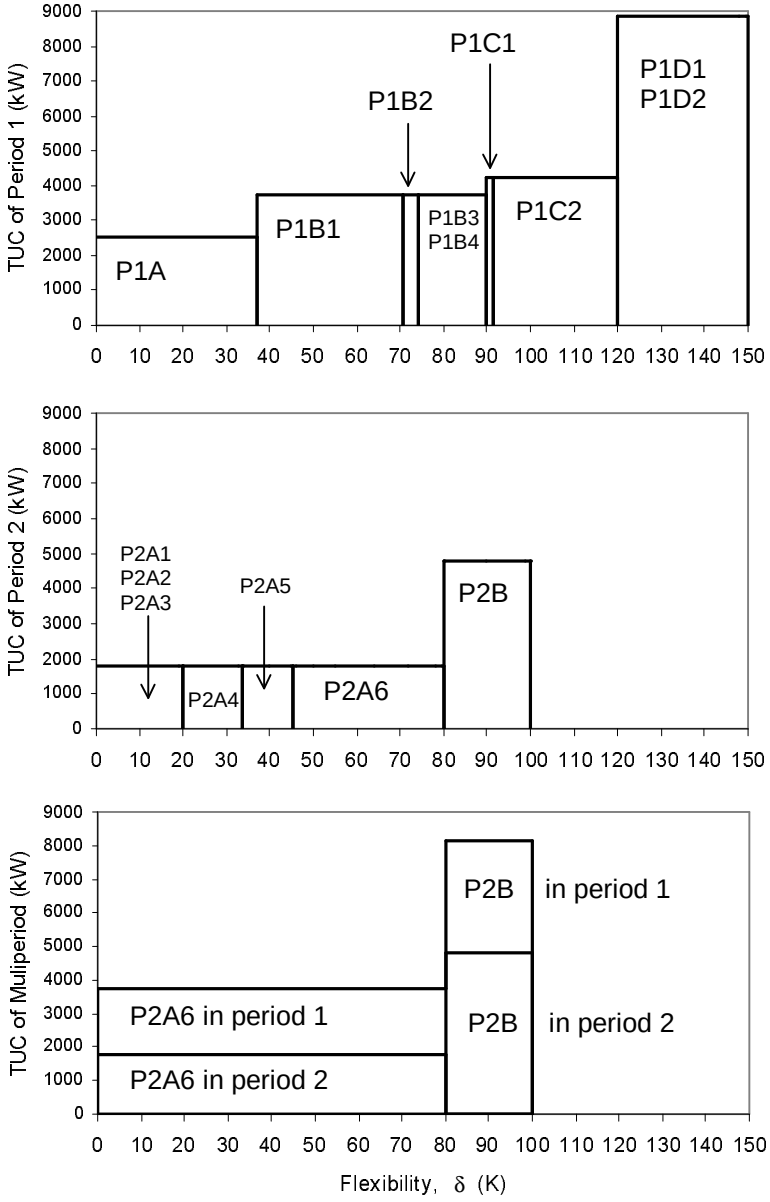


Figure 4. Total-utility consumptions at the nominal operating conditions versus the flexibility ranges of synthesized structures.

The HEN with structure P1A consumes 2550 kW minimum total utility at the nominal operating point, *n=0*, and at the same time tolerates an uncertainty of ±36.95K in the source-stream temperatures. In other words, structure P1A is operable (i.e., it will not violate the outlet-temperature specifications) as long as the deviations in the source-stream temperatures along the vertex directions have magnitudes within $0 \le \delta \le 36.95$ range. In the space of source-stream temperatures, the largest hyper-rectangle that can be inscribed within the feasible region defined by structure P1A has vertices located at ±36.95K as measured from the nominal point at the center. The second plateau, corresponding to 3750 kW minimum total-utility consumption, contains 4 structures, labelled as P1B1-B4. The source-stream-temperature uncertainties that can be tolerated by these structures are as follows: $0 \le \delta_{(P1B1)} \le 70.63$, $0 \le \delta_{(P1B2)} \le 74.24$, and $0 \le \delta_{(P1B3,P1B4)} \le 90$. The third plateau, corresponding to 4250 kW minimum total-utility consumption, contains structures P1C1 and P1C2, which can tolerate source-stream-temperature uncertainties of $0 \le \delta \le 91.51$ and $0 \le \delta \le 120$, respectively. The fourth plateau, corresponding to 8850 kW minimum total-utility consumption, contains structures P1D1 and P1D2, both of which can tolerate source-stream-temperature uncertainties of $0 \le \delta \le 150$.

The HENs with structures P2A1, P2A2, and P2A3 consume 1780 kW minimum total utility at the nominal operating point, *n=0*, and at the same time tolerate an uncertainty of ±20K in their source-stream temperatures. The HEN with structure P2A4 consumes again 1780 kW minimum total utility at the nominal operating point, *n=0*, but it can tolerate an uncertainty of

±33.57K in its source-stream temperatures. The HEN with structure P2A5 also consumes 1780 kW minimum total utility at the nominal operating point, *n=0*, however it can tolerate an uncertainty of ±45.42K in its source-stream temperatures.

The HEN with structure P2A6 can tolerate an uncertainty of ±80K in its source-stream temperatures. P2A6 consumes 3750 kW minimum total utility at the nominal operating point, *n=0*, when it is operating at the first period, *p*=1, but it consumes 1780 kW minimum total utility when it is operating at the second period, *p*=2. Finally, the HEN with structure P2B can tolerate an uncertainty of ±100K in its source-stream temperatures. P2B consumes 8150 kW minimum total utility at the nominal operating point, *n=0*, when it is operating at the first period, *p*=1, but it consumes 4780 kW minimum total utility when it is operating at the second period, *p*=2.

Fig. 4 reveals the trade-off between structural flexibility and minimum total-utility consumption at the nominal operating condition(s). If there is only one favorable structure in a desired flexibility range, then the final design should be done based on that structure. For instance, if a flexibility of 60K is enough for a multiperiod operation, then structure P2A6 is the only choice considering its lower total-utility-consumption level. On the other hand, if there are multiple structures with identical total-utility-consumption levels in a desired flexibility range, then the final decision can be made after computing the total (retrofit) design cost (area and utility costs) for the structures embedded in that flexibility range.

Additional Comments

It should be mentioned that it is inherent in the multiperiod MILP formulation that if an exchanger (match) is present in the reduced structure, then it is present in all periods of operation. In other words, the same units are used in all periods. This may cause overestimation of the total minimum utility consumption in either of the periods. This limitation will be relaxed in further works.

It should also be mentioned that the minimum number of superstructure stages, N_S, should be fixed at $max\{N_H,N_C\}+1$, not at $max\{N_H,N_C\}$ as suggested by Yee and Grossmann (1990a), in order to capture other possible structures that may show lower total utility consumptions.

The pinch (minimum utility) solution (Linnhoff and Flower, 1978a, 1978b) of the first period, *p*=1, with two superstructure stages and with $NEU_{max}=6$ (Eq.37) is identical to structure P1A (2550 kW total utility consumption). However, the pinch solution of the second period, *p*=2, can be obtained only with three superstructure stages and with $NEU_{max}=7$. Also, the pinch solution of the multiperiod problem, *P*=2, can be obtained only with three superstructure stages and with $NEU_{max}=9$. Therefore, the results presented in this work should be considered as a demonstration of the multiperiod MILP formulation. The true minimum-utility results are not shown in this work due to space limitations; since with three superstructure stages and with $NEU_{max}=9$ too many structures are obtained.

Determination of Structure Boundaries

The determination of the structure boundaries (the determination of the flexibility indexes, δ_p^* of the synthesized structures), for instance, in the construction of Fig. 4, may be difficult if the

MILP formulation is solved consecutively for increasing values of the desired flexibility, δ_p. This difficulty is due to the possible presence of multiple structures with identical minimum total-utility-consumption levels at a given value of the desired flexibility. The determination of the structure boundaries, δ_p^*, can be formulated as the *flexibility-index* problem, i.e., finding the maximum value of the flexibility, $\delta=\delta_p^*$, that a given structure (fixed binary variables) can tolerate. For fixed heat-capacity-flowrates, the flexibility-index problem is a LP problem that involves linear equality constraints corresponding to the energy balances for the process/process and process/utility matches, and linear inequality constraints to guarantee positive driving forces for the temperatures.

Alternatively, instead of formulating the flexibility-index problem for each structure encountered in the solution range of the MILP formulation, the equations of the MILP formulation can be used directly after removing the binary variables from the set of decision variables and by introducing them as constant parameters. This approach is advantageous as it requires only minor modifications in the formulation (and in the GAMS coding) of the MILP problem. The objective function of the flexibility-index LP problem for a fixed structure is given by Eq.41, and the required equality and inequality constraints are as given before by Eqs.(12-39).

$$\max_{\substack{t_{i,k,n,p},\, t_{j,k,n,p},\\ q_{i,j,k,n,p}, qcu_{i,n,p}, qhu_{j,n,p},\\ dt_{i,j,k,n,p}, dtcu_{i,n,p}, dthu_{j,n,p}}} \sum_{p\in P} \delta_p \qquad k\in ST,\ n=0,1,...,N \qquad (41)$$

For a structure obtained from the solution of the MILP formulation (Eqs.11-39), the solution, δ_p^*, of the flexibility-index LP problem, Eqs.(41, 12-39) gives the flexibility index, and thus, the exact region of validity of that structure.

Conclusions

The synthesis of multiperiod minimum-utility HEN structures satisfying given period-specific flexibility targets was accomplished through one-stage, non-iterative, superstructure-based-simultaneous-MILP formulation. The formulation presumed that the feasible region defined by the reduced inequality constraints was convex in uncertain parameters, and thus the optimal solution was explored on the basis of the vertices of the polyhedral region of uncertainty. The superstructure-based formulation was used also to compute the vertex-based flexibility indexes the synthesized structures. The solution of the superstructure-based flexibility-index formulation determined the structural validity regions over which the structures were operable with the corresponding minimum utility-consumption levels.

The significance of the “simultaneous flexibility targeting and multiperiod synthesis” formulation for minimum-utility multiperiod HEN structures is that the flexibility of the synthesized structure provides an upper bound for the flexibility that should be expected from the design or retrofit design of the structure. The value of the objective function provides a lower bound for the total-utility-consumption level that should be expected from the design or retrofit design having a flexibility equal to the structural flexibility targeted in the multiperiod synthesis.

The basic features of the "simultaneous flexibility targeting and multiperiod synthesis" formulation were demonstrated using a well-known HEN superstructure of Yee and Grossmann. An example two-period HEN problem was solved successively over a range of flexibility targets by considering the source-stream temperatures as the uncertain parameters. The trade-off between the flexibilities and the total-utility-consumption levels for the reduced multiperiod structures accommodated in the desired flexibility range were revealed.

Acknowledgements

Financial supports provided by Gebze Institute of Technology Research Foundation (Project No: 99B030602), TUBITAK (The Scientific and Technical Research Council of Turkey) (Project No: MISAG-83), and Bogazici University Research Foundation (Project No: 99A503) are gratefully acknowledged.

Nomenclature

CP = index set of cold process stream
d = vector of design variables
$dt_{i,j,k,n}$ = temperature approach for match i and j in stage k at vertex n
$dtcu_{i,n}$ = temperature approach for match i and cold utility at vertex n
$dthu_{j,n}$ = temperature approach for match j and hot utility at vertex n
F = heat-capacity-flowrate
g = vector of inequality constraints
h = vector of equality constraints
HP = index set of hot process stream
n = vertex-direction identifier (subscript or superscript)
N = total number of vertices
N_C = number of cold streams
N_H = number of hot streams
N_S = number of superstructure stages
NEU_{max} = maximum number of heat-exchange units allowed
p = vector of fixed parameters
P = number of periods
$q_{i,j,k,n}$ = heat exchanged between stream i and j in stage k at vertex n
$qcu_{i,n}$ = heat exchanged between stream i and cold utility at vertex n
$qhu_{j,n}$ = heat exchanged between stream j and hot utility at vertex n
$\mathbf{r}_o$ = operating-point identifier at the nominal operating condition
$\mathbf{r}_n$ = operating-point identifiers for vertices as N combinations of –1 and +1
R = region describing the scaled hyperrectangle
ST = index set of superstructure stages
$t_{i,k,n}$ = temperature of stream i at hot end of stage k at vertex n
$t_{j,k,n}$ = temperature of stream j at hot end of stage k at vertex n
T = temperature
ΔT_{min} = minimum approach temperature
u = vector of free/control variables
x = vector of state variables
z = vector of binary variables
$z_{i,j,k}$ = binary variable for existence of unit for match i and j in stage k

zcu_i = binary variable for existence of unit for match i and cold utility in stage k
zhu_j = binary variable for existence of unit for match j and hot utility in stage k

Greek Letters

α = period weighting parameter used in the objective function
δ = flexibility (scalar)
δ^* = flexibility index (scalar)
δ_D^* = design flexibility index
δ_{RD}^* = retrofit-design flexibility index
Φ = objective function
Γ = upper bound for temperature difference
θ = vector of uncertain parameters
$\Delta\theta^-$, $\Delta\theta^+$ = scaled deviations from θ^o
Ω = upper bound for heat exchange

Subscripts

CU = cold utility
HU = hot utility
i = hot process stream
j = cold process stream
k = index for superstructure stage
p = period identifier

Superscripts

in = inlet
o = value at nominal operating condition
out = outlet

References

Bhatia T.K., Biegler L.T. (1999). "Multiperiod Design and Planning with Interior Point Methods", *Comp. Chem. Eng.*, Vol.23, No.7, pp.919-932.

Bok J.K., Grossmann I.E., Park S. (2000). "Supply Chain Optimization in Continuous Flexible Process Networks", *Ind. Eng. Chem. Res.*, Vol 39, No.5, pp.1279-1290.

Brooke A., Kendrick D., Mearaus A. (1992). *GAMS: A User's Guide*; Scientific Press, Palo Alto, California.

Dedopoulos I.T., Shah N. (1996). "Long-Term Maintenance Policy Optimization in Multipurpose Process Plants", *Trans. IChemE*, Vol.74, Part A, pp.307-320.

Floudas C.A. (1995). *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications*, Oxford Univ. Press; New York.

Halemane K.P., Grossmann I.E. (1983). "Optimal Process Design under Uncertainty", *AIChE J.*, Vol.29, No.3, pp.425-433.

Iyer R.R., Grossmann I.E. (1997). "Optimal Multiperiod Operational Planning for Utility Systems", *Comp. Chem. Eng.*, Vol.21, No.8, pp.787-800.

Konukman A.E.S., Camurdan M.C., Akman U. (1999). "Synthesis of Energy-Optimal HEN Structures with Specified Flexibility Index through Simultaneous MILP Formulation",

PRES'99: Proceedings of the 2 nd Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction, pp.395-400, (Editors: F. Friedler, J. Klemes), Hungarian Chemical Society, Budapest, Hungary.

Linnhoff B., Flower J.R. (1978a). "Synthesis of Heat Exchanger Networks: I. Systematic Generation of Energy Optimal Networks", *AIChE J.*, Vol.24, pp.633-642.

Linnhoff B., Flower J.R. (1978b). "Synthesis of Heat Exchanger Networks: II. Evolutionary Generation of Networks with Various Criteria of Optimality", *AIChE J.*, Vol.24, pp.633-642.

Papalexandri K.P., Pistikopoulos E.N. (1993). "An MINLP Retrofit Approach for Improving the Flexibility of Heat Exchanger Networks", *Ann. Oper. Res.*, Vol.42, pp.119-168.

Papalexandri K.P., Pistikopoulos E.N. (1994). "A Multiperiod MINLP Model for the Synthesis of Flexible Heat and Mass Exchange Networks", *Comp. Chem. Eng.*, Vol.18, No.11/12, pp.1125-1139.

Paules G.E. IV, Floudas C.A. (1992). "Stochastic Programming in Process Synthesis: A Two-Stage Model with MINLP Recourse for Multiperiod Heat-Integrated Distillation Sequences," *Comp. Chem. Eng.*, Vol.16, No.3, pp.189-210.

Rooney W.C., Biegler L.T. (1999). "Incorporating Joint Confidence Regions into Design under Uncertainty", *Comp. Chem. Eng.*, Vol 23, No.10, pp.1563-1575.

Sahinidis N.V., Grossmann I.E., Fornari R.E., Chathrathi M. (1989). "Optimization Model for Long Range Planning in the Chemical Industry", *Comp. Chem. Eng.*, Vol.13, No.9, pp.1049-1063.

Swaney R.E., Grossmann I.E. (1985). "An Index for Operational Flexibility in Chemical Process Design - Part I: Formulation and Theory", *AIChE J.*, Vol.31, No.4, pp.621-630.

Van den Heever S.A., Grossmann I.E. (1999). "Disjunctive Multiperiod Optimization Methods for Design and Planning of Chemical Process Systems", *Comp. Chem. Eng.*, Vol 23, No.8, pp.1075-1095.

Varvarezos D.K., Grossmann I.E., Biegler L.T. (1992). "An Outer-Approximation Method for Multiperiod Design Optimization", *Ind. Eng. Chem. Res.*, Vol.31, No.6, pp.1466-1477.

Varvarezos D.K., Biegler L.T., Grossmann I.E. (1994). "Multiperiod Design Optimization with SQP Decomposition", *Comp. Chem. Eng.*, Vol.18, No.7, pp.579-595.

Varvarezos D.K., Biegler L.T., Grossmann I.E. (1995). "Modeling Uncertainty and Analyzing Bottleneck Characteristics in Multiperiod Design Optimization", *Comp. Chem. Eng.*, Vol.19, No.5, pp.497-511.

Yee T.F., Grossmann I.E. (1990a). "Simultaneous Optimization Models for Heat Integration – I. Area and Energy Targeting and Modeling of Multi-Stream Exchangers", *Comp. Chem. Eng.*, Vol.14, No.10, pp.1151-1164.

Yee T.F., Grossmann I.E. (1990b). "Simultaneous Optimization Models for Heat Integration – II. Heat Exchanger Network Synthesis", *Comp. Chem. Eng.*, Vol.14, No.10, pp. 1165-1184.